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Finite-time synchronization of multi-manipulator systems under aperiodically intermittent communication

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Abstract. This paper studies the finite-time synchronization control problem of multi-manipulator systems under aperiodically intermittent communication. In contrast to the traditional periodic intermittent communication schemes widely adopted in previous research, the introduction of aperiodic patterns breaks through the limitations of regular communication intervals. This novel approach more realistically mimics the complex and unpredictable communication conditions often encountered in actual industrial and robotic applications. By employing the appropriate auxiliary variables and establishing a velocity estimator, a coordinated tracking controller is designed to realize the finite-time synchronization. Note that the settling time monotonically increases with the maximum rest ratio that the system can tolerate. Finally, the validity of proposed finite-time synchronization strategies is verified through a numerical simulation.

Keywords: Finite-time / multi-manipulator systems / aperiodically intermittent communication / coordinated tracking control

1 Introduction

With the booming economy and the industrial revolution, multi-agent systems (MASs) have garnered significant attention owing to their application across various industries. Compared with traditional single-agent systems, MASs are more flexible and reliable. For example, compared to a single manipulator, multi-manipulator systems can perform more complex tasks, including heavy-load lifting, item repairing, collaborative object manipulation, etc. [1–3]. However, most practical systems exhibit nonlinear characteristics. Hu et al. [4] investigated the conflict prevention path planning problems of AGVs. Additionally, MASs find widespread applications in practical systems such as driverless cars [5], drones [6], mechanical arms [7], etc. Particularly, a part of nonlinear systems can be elucidated using the Euler-Lagrange equation. This equation can better handle feedback for the velocity of the manipulators, but because of its uncertainties and nonlinearity, the results derived from linear MASs can not be directly applicable to guarantee the synchronization of Euler-Lagrange systems. Nevertheless, note from [8] that the synchronization and containment problems of the multiple uncertain Euler-Lagrange systems could be solved. The model of multi-manipulator systems is determined by Euler Lagrange equations.

Even if the multi-manipulator systems have many advantages, they are still susceptible to various factors such as communication interruptions, equipment damage, etc. Intermittent communication is more prevalent in practice than other factors. Thus, in view of this issue, a distributed coordinated tracking control strategy was designed as described in references [9]. To ensure that the multi-robot systems don't lose mission-critical data under intermittent communications, a multi-layer networking solution was designed [10]. The intermittent communications considered above are all periodic, which involve fixed interruption of communication time and rest time. However, this does not accurately reflect the intermittent communication that exists in reality. The synchronization problem under aperiodic intermittent communication has been studied [11–13]. Taking the discrete-time stochastic delayed into account, the stabilization of neural networks by intermittent control was investigated [11]. The synchronization of second-order MASs under aperiodically intermittent communication was reached by designing a sampled-hold-based intermittent communication [12]. Furthermore, considering the problem of nonlinear delayed MASs under aperiodically intermittent communication, the author proposed a leader-following controller to solve the problem [14]. The study of aperiodically intermittent communication in multi-manipulator systems has been limited, underscoring the significance of our research work.

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In the study of MASs, it is inevitable to discuss the synchronization problem. Both the literature [15,16] have provided many meaningful results [15,16]. Reference [15] reviewed advances in multi agent system coordination and analyzed key approaches and open challenges. Reference [16] reviewed recent advances in constrained multi-agent consensus and coordination, categorizing key approaches and their applications to smart grids and clustering algorithms. In facing actual task requirements, we aim for the systems to reach a synchronization state within a limited time so that the subsequent work can proceed. Therefore, a finite-time controller is necessary to control the entire system. Consequently, a finite-time strategy tailored for heterogeneous MASs was developed [17]. Du et al. [18] studied the finite-time synchronization problem of MASs under directed topology using an event-triggered strategy. According to the homogeneous theorem, references [19–21] investigated the synchronization problems of multi-agents and complex networks in finite-time [19–21]. However, the systems discussed above are linear systems and in actual production, nonlinear systems are more common. Therefore, in recent years, attention has gradually shifted towards nonlinear systems. For example, a smooth adaptive finite-time strategy was proposed for multiple Euler-Lagrange systems, addressing systems uncertainties and external disturbances [22]. In [23], a finite-time protocol is proposed for the coordinated behavior of multiple Euler-Lagrange systems in cooperative competitive networks [23]. To address the problem of finite-time cooperative control inherent in multi-manipulator systems, Van et al. [24] optimized control method using integral sliding mode control and improved robustness to issues including systems interference and model uncertainty. However, there exists a scarcity of research addressing finite-time synchronization under aperiodically intermittent communication.

In this paper, we investigate the finite-time synchronization control problem of multi-manipulator systems under aperiodically intermittent communication. The main work and contributions of this paper are as follows.

- A new finite-time estimator is established to estimate unknown velocities. In addition, for unknown velocities of Euler-Lagrange systems, the estimator is designed to obtain a number of sufficient conditions that guarantee finite-time synchronization of multi-manipulator systems.
- For multi-manipulator systems under intermittent communication, a new distributed finite-time synchronization strategy is designed. Different from intermittent communications in [10], our considered intermittent communication scheme is completely aperiodic.
- Several novel criteria are obtained to guarantee finite-time synchronization of multi-manipulator systems under aperiodically intermittent communication. Moreover, the settling time can be accurately estimated.

Notations: Through this paper, for a given scalar $l>0$, $sig(x)^l = [sgn(x_1)|x_1|^l, sgn(x_2)|x_2|^l, \cdots, sgn(x_n)|x_n|^l]^T$ and sgn $(\bullet)$ means the signal function. $\|\bullet\|$ denotes the Euclidean vector norm or its induced matrix two-norm. $\otimes$ stands for Kronecker product.

2 Preliminaries and problem formulations

2.1 Algebraic graph theory

In the paper, the communication topology for multi-manipulator systems is set as an undirected graph. Let $\mathscr{G}=\{\mathcal{V}, \varepsilon, \mathscr{A}\}$, where the node set is denote by $\mathcal{V}=\{1, 2, \cdots, N\}$, the edge set $\varepsilon\subset\mathcal{V}\times\mathcal{V}$ and $\mathscr{A}=[a_{ij}]_{N\times N}$ is the weighted adjacency matrix. Define as an edge ε_{ij} in ε and $\varepsilon_{ij}=(v_i, v_j)$. The signed weight $a_{ij}=1$, if the information from the node jth can be received by the node ith. Otherwise, $a_{ij}=0$. Furthermore, $a_{ii}=0$. The neighbors of node i are defined as $\mathcal{N}_i=\{j|a_{ij}\neq 0\}$. Define the degree matrix $\mathscr{D}=\{d_1, d_2, d_3, \cdots, d_N\}$. The Laplacian matrix for the graph $\mathscr{G}$ is $\mathscr{L}=\mathscr{D}-\mathscr{A}$, and $\mathscr{L}=[l_{ij}]\in\mathscr{R}^{N\times N}$ is described as

$$l_{ii}=\sum_{j=1,j\neq i}^{N}|a_{ij}|, l_{ij}=-a_{ij}, i\neq j.$$

There exists node 0 that is assigned as the leader system. And then the graph $\tilde{\mathrm{G}}$ is defined the graph with the leader node included. So, the communication topology involves $N+1$ nodes, and the matrix $\mathscr{H}=\mathscr{L}+G$ is denoted, where $G=diag\{a_{i0}, a_{i1}, a_{i2}, \cdots, a_{iN}\}$, $i=1, 2, 3,\cdots, N$. If the information from leader node 0 can be passed to the i_{th} follower, the weighting gain $a_{i0}=1$, otherwise, $a_{i0}=0$.

Assumption 2.1. For the graph $\tilde{\mathscr{G}}$, there exists at least one path between any two nodes.

2.2 Problem formulation

The multi-manipulator systems are represented as

$$M_i(x_i)\ddot{x}_i + C_i(x_i,\dot{x}_i)\dot{x}_i + G_i(x_i) = \tau_i(t), \tag{1}$$

where $i=1, 2, \cdots, N$, $x_i, \dot{x}_i$ and $\ddot{x}_i\in\mathscr{R}^n$ describe the state vector of the ith position, velocity and acceleration of every mechanical arm. $M_i(x_i)\in\mathscr{R}^{n\times n}$ is a positive definite inertia matrix, $C_i(x_i,\dot{x}_i)\in\mathscr{R}^{n\times n}$ is Coriolis and centripetal forces matrix, $G_i(x_i))\in\mathscr{R}^n$ is the gravity vector, $\tau_i(t)\in\mathscr{R}^n$ is generalized force vector.

The system (1) can be rewritten as

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} + G(x) = \tau(t), \tag{2}$$

where $M(x)=diag\{M_1(x_1), M_2(x_2),\cdots, M_n(x_n)\}$, $C(x, \dot{x}) = diag\{C_1(x_1, \dot{x}_1), C_2(x_2, \dot{x}_2),\cdots, C_n(x_n, \dot{x}_n)\}$, $G(x) = (G_1^T(x_1), G_2^T(x_2), \cdots, G_n^T(x_n))^T$, $x=({x_1}^T, {x_2}^T, \cdots, {x_n}^T)^T$, $\tau(t)=({\tau_1}^T(t), {\tau_2}^T(t), \cdots, {\tau_n}^T(t))^T$.

The leader is given by the following

$$\dot{x}_0 = Q, \tag{3}$$

where Q is a constant.

The error systems are defined as follows

$$\begin{cases} e_{qi} = x_i - x_0 \\ e_{vi} = \dot{x}_i - \dot{x}_0 \end{cases} \tag{4}$$

where $e_{qi}=0$ and $e_{vi}=0$.

According to (1) and $\ddot{x}_0 = 0$,

$$\begin{cases} \dot{e}_x = e_v \\ \dot{e}_v = M(q)^{-1}[\tau(t) - C(x, \dot{x}) - G(x)] \end{cases}. \tag{5}$$

The system (1) is characterized by the following properties:

Property 2.1. [25] $\dot{M}_i(x_i) - 2C_i(x_i, \dot{x}_i)$ represents a skew-symmetric matrix.

Property 2.2. [25] $k_{\underline{m}}, k_{\overline{m}}$, k_c and k_g are defined as positive constants. For $\forall i \in \mathcal{V}$, $k_{\underline{m}} I_p \leq M_i(x_i) \leq k_{\overline{m}} I_p$, $\|C_i(x_i, \dot{x}_i)\| \leq k_c \|\dot{x}_i\|$ and $\|G_i(\overline{x_i})\| \leq k_g$.

Definition 2.1. Under Assumption 1, multi-manipulator systems are regarded to have reached synchronization in finite-time t_0 if the following conditions can be met,

$$\begin{aligned} &\lim_{t \to t_0} ||x_i(t) - x_0(t)|| = 0 \\ &\lim_{t \to t_0} ||\dot{x}_i(t) - \dot{x}_0(t)|| = 0 \end{aligned} \tag{6}$$

where $i = 1, 2, 3, \cdots, N$ and $t_0 > 0$.

Assumption 2.2. The generalized velocity and acceleration of the leader do not grow infinitely.

Lemma 2.1. [26] For $\forall x, y \in \mathscr{R}^n$ and $\mathscr{I} > 0$, one has $2x^T y \leq \mathscr{I} x^T x + \mathscr{I}^{-1} y^T y$.

Lemma 2.2. [27] There exist a Laplacian matrix L and a diagonal matrix$B$. If Assumption 1 holds and the graph doesn't have nonnegative weights, then there exists a positive diagonal matrix $F = diag(f_1, f_2, f_3, \cdots, f_n)$, where $f_i = \frac{c_i}{d_i}$, $c = [c_1, c_2, c_3, \cdots, c_n]^T = (L+B)^{-1} 1_N$, $d = [d_1, c_2, d_3, \cdots, d_n]^T = (L+B)^{-T} 1_N$, and it satisfied $U = FH + H^T F > 0$, where $H = L + B$.

Definition 2.2. [28] For aperiodically intermittent communication, a constant $0 \leq \varepsilon < 1$ is defined as follows, for $m = 0, 1, 2, 3, \cdots$

$$\varepsilon = \lim_{m \to \infty} sup \frac{t_{m+1} - s_m}{t_{m+1} - t_m}. \tag{7}$$

The communication situation with intermittent communication consists of the normal communication time $T_m = [t_m, s_m)$ and the intermittent communication time $T_s = [s_m, t_{m+1})$. Clearly, aperiodically intermittent communication transitions to continuous communication when ε=0.

Assumption 2.3. There exist two positive scalars $0 < \rho < \mu < +\infty$, for $m = 0, 1, 2, \cdots$,

$$\begin{aligned} &\inf_m (s_m - t_m) = \rho \\ &\sup_m (t_{m+1} - t_m) = \mu \end{aligned}. \tag{8}$$

As defined in equation (8), it can be seen that ρ represents the minimum permitted communication time in aperiodic intermittent communication, while μ indicates the maximum interruption time of communication.

Lemma 2.3. [29] Assume $z_i \in \mathbb{R}_+$, $i = 1, 2, 3, \cdots, \ell$ and $\aleph \in (0, 1)$, then

$$\sum_{i=1}^{\ell} |z_i|^{\aleph} \geq (\sum_{i=1}^{\ell} |z_i|)^{\aleph}. \tag{9}$$

Lemma 2.4. [29] For $q_1, q_2, q_3, \cdots, q_n \in R^n$ and $0 < p < 2$, then

$$\begin{aligned} &||q_1||^p + ||q_2||^p + ||q_3||^p + \cdots \\ &\quad + ||q_n||^p \geq (||q_1||^2 + ||q_2||^2 + ||q_3||^2 + \cdots + ||q_n||^2)^{\frac{p}{2}}. \end{aligned} \tag{10}$$

Lemma 2.5. [30] Suppose that there exist a continuous system $\dot{s} = f(s)$ which $f(0) = 0$, a positive definite function $V(t)$ and its neighborhood $U \in R^{n \times N}$ at the origin satisfy the following equations

$$\begin{cases} \dot{V}(t) \leq -\gamma V^{\eta}(t) & t \in T_m \\ \dot{V}(t) \leq 0 & t \in T_s \end{cases} \tag{11}$$

so that $V(t) \equiv 0$, when $t \geq T^*$ and the setting time satisfies

$$T^* \leq \frac{V^{1-\eta}(x(t_0), t_0)}{\gamma(1-o)(1-\eta)}. \tag{12}$$

Lemma 2.6. [31] Assume a continuous and positive definite function $V(t)$, and the following equations can be satisfied:

$$\begin{cases} \dot{V}(t) \leq -v_1 V(t) - v_2 V^p(t) & t \in T_m \\ \dot{V}(t) \leq v_3 V(t) & t \in T_s \end{cases} \tag{13}$$

where v_1, $v_2 \geq 0$, $0 < p < 1$ then

$$\begin{aligned} V^{1-p}(t) e^{(1-p)v_1 t} \leq\ & e^{(1-p)(v_1+v_3)\varepsilon t} (V^{1-p}(0) + \frac{v_2}{v_1} \\ & - \frac{v_2}{v_1} e^{(1-p)v_1 t} e^{-(1-p)(v_1+v_3)\varepsilon t}). \end{aligned} \tag{14}$$

The settling time T satisfies

$$T \leq \frac{1}{1-p} \frac{ln(1 + \frac{v_1}{v_2} V^{1-p}(0))}{(v_1 - (v_1 + v_3)\varepsilon)}. \tag{15}$$

2.3 Control design

A finite-time control protocol under aperiodic intermittent communication will be designed. Two auxiliary variables are defined by the following equations

$$\begin{cases} \phi_i(t) = -k_1 \sum_{j=0}^{n} a_{ij}(x_i(t) - x_j(t)) - k_2 \sum_{j=0}^{n} a_{ij} sig(x_i(t) - x_j(t))^{\alpha}, & t \in T_m \\ \phi_i(t) = 0, & t \in T_s \end{cases} \tag{16}$$

$$\begin{cases} \psi_i(t) = -k_3\sum_{j=0}^{n} a_{ij}(s_i(t) - s_j(t)) - k_4\sum_{j=0}^{n} a_{ij}sig(s_i(t) - s_j(t))^{\alpha}, & t\in T_m \\ \psi_i(t) = 0, & t\in T_s \end{cases} \tag{17}$$

where $0 < \alpha < 1$, k_1, k_2, k_3 and k_4 are positive constants that will be designed later, s_i represents the virtual error of the *ith* follower and satisfies the following equation

$$s_i = \dot{x}_i - \hat{v}_i - \phi_i(t) \tag{18}$$

here $\hat{v}_i$ is the estimates for the *ith* follower,

$$\begin{cases} \dot{\hat{v}}_i(t) = -y_1[\sum_{j=1}^{n} b_{ij}(\hat{v}_i(t) - \hat{v}_j(t)) + b_{i0}(\hat{v}_i(t) - \dot{x}_0(t))]^u, & t\in T_m \\ \dot{\hat{v}}_i(t) = 0, & t\in T_s \end{cases} \tag{19}$$

where $y_1 > 0$, $0 < u < 1$, and $\hat{A} = [b_{ij}]_{N \times N}$ is the adjacency matrix of an undirected graph $\mathscr{G}_{\mathscr{B}}$ describing the information exchange between N followers of $\hat{V}_i$, and if there is a path between the leader x_0 and the *i*th follower, $b_{i0} = 1$, otherwise, $b_{i0} = 0$. The Laplacian matrix L_B for graph $\mathscr{G}_{\mathscr{B}}$ is defined by $L_B = \mathscr{D}_B - \hat{A}$, where $\mathscr{D}_{\mathscr{B}}$ is the degree matrix.

Remark 3.1. Due to environmental disturbances, equipment damage, and network congestion, the systems cannot maintain continuous communication. Consequently, the control protocols must be designed differently for intermittent communication compared to continuous communication.

3 Results

3.1 The finite-time synchronization of estimator under aperiodic intermittent communication

In this part, we delve into the finite-time synchronization of the designed estimator.

Theorem 1. Suppose that the graph $\tilde{\mathscr{G}}_B$ satisfies Assumption 1 and Assumption 2, then $\|\hat{v}_i - \dot{x}_0\|$ converages to zero within a finite-time T_1, where T_1 satisfies

$$T_1 \le \frac{(1+u)V^{\frac{1-u}{1+u}}(0)}{c_1(1-\varepsilon)(1-u)}.$$

Proof. When $t\in T_m$, the following equation can be obtained

$$\begin{aligned} \dot{\overline{v}}_i(t) &= \dot{\hat{v}}_i(t) - \ddot{x}_0(t) \\ &= -y_1[\sum_{j=1}^{n} b_{ij}(\overline{v}_i(t) - \overline{v}_j(t)) + b_{i0}(\overline{v}_i(t)]^u \\ &= -y_1[\sum_{j=0}^{n} b_{ij}(\overline{v}_i(t) - \overline{v}_j(t))]^u \\ &= -y_1[(H_B \otimes I_m)\overline{v}_i(t)]^u. \end{aligned}$$

And then, let $\tilde{v}_i(t) = (H_B \otimes I_m)\overline{v}_i(t)$,

$$\dot{\tilde{v}} = (H_B \otimes I_m)(-y_1(\tilde{v})^u) \tag{20}$$

where $\tilde{v} = [\tilde{v}_1^T, \tilde{v}_2^T, \tilde{v}_3^T, \cdots, \tilde{v}_n^T]^T$, $\mathscr{H}_{\mathscr{B}} = L_B + diag(b_{10}, b_{20}, b_{30}, \cdots, b_{n0})$. If the graph $\mathscr{G}_{\mathscr{B}}$ satisfies Assumption 1, then the eigenvalues of $\mathscr{H}_{\mathscr{B}}$ are positive. And a symmetric positive definite matrix $\mathscr{F} \in \mathscr{R}^{N\times N}$ satisfied $\mathcal{U} = \mathscr{F}\mathscr{H}_{\mathscr{B}} + \mathscr{H}_{\mathscr{B}}{}^T\mathscr{F} > 0$.

The Lyapunov candidate function is considered as

$$V_0 = \sum_{i=1}^{n}\sum_{l=1}^{N} \frac{f_i}{1+u}\tilde{v}_{il}^{1+u} \tag{21}$$

where f_i is as same as Lemma 2.

The derivative of V_0 is

$$\begin{aligned} \dot{V}_0 &= \sum_{i=1}^{n}\sum_{l=1}^{N} f_i\tilde{v}_{il}^u\dot{\tilde{v}}_{il} \\ &= \sum_{i=1}^{n} f_i(\tilde{v}_i^u)^T\dot{\tilde{v}}_i \quad . \end{aligned}$$

Combining with Lemma 2, it can be deduced that

$$\begin{aligned} \dot{V}_0 &\le -y_1(\tilde{v}^u)^T(FH \otimes I_n)(\tilde{v}^u) \\ &\le -\frac{1}{2}y_1\lambda_{min}(U)\sum_{i=1}^{n}\sum_{l=1}^{N}\tilde{v}_{il}^{2u}. \end{aligned} \tag{22}$$

Based on Lemma 4, one has

$$\begin{aligned} \dot{V}_0 &\le -\frac{1}{2}y_1\lambda_{min}(U)(\frac{1+u}{f_{max}})^{\frac{2u}{1+u}}V_0^{\frac{2u}{1+u}} \\ &= -c_1V_0^{\frac{2u}{1+u}} \end{aligned} \tag{23}$$

where $c_1 = -\frac{1}{2}y_1\lambda_{min}(U)\left(\frac{1+u}{f_{max}}\right)^{\frac{2u}{1+u}}$, $f_{max} = max\{f_1, f_2, f_3, \cdots, f_n\}$.

When $t\in T_s$, $\tilde{v} = 0$ is clearly visible. It is obviously that the Lyapunov candidate function of $\tilde{v}$ is less than or equal to 0, which makes $\dot{V}_0 \le 0$.

Via Lemma 4, the estimator converges to 0 in a finite-time $T_1 \le \frac{(1+u)V^{\frac{1-u}{1+u}}(0)}{c_1(1-\varepsilon)(1-u)}$.

Remark 3.2. Sometimes it is hard to get the exact speed of the leader in actual multi-manipulator systems. Therefore, we designed a new estimator for each manipulator system to estimate the unmeasurable velocities under aperiodically intermittent communication.

3.2 Finite-time synchronization under aperiodically intermittent communication

Substituting (18), (17) and (15) into (5), the following equations are obtained

$$\begin{cases} \dot{e}_i(t) = s_i(t) + \phi_i(t) + \overline{v}_i t \\ \dot{s}_i(t) = M_i(x_i)^{-1}[\tau_i(t) - C_i(x_i, . \dot{x}_i) - G_i(x_i)] - \dot{\phi}_i(t) - \dot{\overline{v}}_i(t), \end{cases} \tag{24}$$

where $e_i(t) = x_i(t) - x_0(t)$, $\overline{v}_i t = \hat{v}_i(t) - \dot{x}_0(t)$.

Then, the following control protocols are proposed:

$$\tau_i(t) = M_i(x_i)(\psi_i + \dot{\phi}_i + y\phi_i - aS_i) - yM_i(x_i)(x_i - \hat{v}_i) + C_i(x_i, \dot{x}_i)\dot{x}_i + G_i x_i \quad (25)$$

where a and y are positive scalars.

Theorem 2. Suppose the graph $\mathscr{G}$ satisfies Assumption 1, and there are positive constants $\mathscr{I}$, l_1 and l_2 satisfying the following inequalities

– $I - 2k_1\lambda_{min}(\mathscr{H}) \le -l_1$
– $I^{-1} - 2k_3\lambda_{min}(\mathscr{H}) - 2(a+y) \le -l_1$
– $I^{-1} - 2(a+y) \le l_2$
– $I \le l_2$
– $l_1 - (l_1 + l_2)\varepsilon > 0$.

Then, the systems (1) can achieve synchronization within a finite-time T under the designed control protocols (25).

$$T \le T_1 + \frac{2}{1-\alpha}\frac{ln(1 + \frac{l_1}{k}\lambda_{min}(H)V^{\frac{1-\alpha}{2}}(0))}{(l_1 - (l_1 + l_2)\varepsilon)}. \quad (26)$$

Proof. Combining (20) and (16) yields the following systems

$$\begin{cases} \dot{e}_{xi} = s_i + \phi_i + \overline{v}_i \\ \dot{s}_i = \psi_i - (a+y)s_i - \dot{\overline{v}}_i - y\overline{v}_i. \end{cases} \quad (27)$$

Consider the candidate Lyapunov function

$$V = e^T e + s^T s \quad (28)$$

Step 1: When $t \in T_m$, the derivation of the Lyapunov function becomes

$$\begin{aligned} \dot{V} &= 2e^T(s + \phi + \overline{v}) + 2s^T(\psi - (a+y)s - \dot{\overline{v}} - y\overline{v}) \\ &= 2e^T s + 2e^T\phi + 2s^T\phi - 2(a+y)s^T s - 2ys^T\dot{\overline{v}} \\ &\le Ie^T e + I^{-1}s^T s - 2(a+y)s^T s \\ &- 2ys^T[(H_B \otimes I_n)\overline{v}_i(t)]^u - 2k_1 e^T\sum_{i=1}^{n}\sum_{j=0}^{n} a_{ij}(x_i(t) \\ &- x_j(t)) - 2k_2 e^T\sum_{i=1}^{n}\sum_{j=0}^{n} a_{ij}sig(x_i(t) - x_j(t))^\alpha \\ &- 2k_3 s^T\sum_{i=1}^{n}\sum_{j=0}^{n} a_{ij}(s_i(t) - s_j(t)) \\ &- 2k_4 s^T\sum_{i=1}^{n}\sum_{j=0}^{n} a_{ij}sig(s_i(t) - s_j(t))^\alpha \end{aligned}$$

$$\begin{aligned} &\le (I - 2k_1\lambda_{min}(H))e^T e + (I^{-1}2k_3\lambda_{min}(H) - 2(a+y))s^T s \\ &\quad - 2k_2\sum_{i=1}^{n}\sum_{j=0}^{n} a_{ij}e_i^T sig(e_i(t) - e_j(t))^\alpha \\ &-2k_4\sum_{i=1}^{n}\sum_{j=0}^{n} a_{ij}s_i^T sig(s_i(t) - s_j(t))^\alpha \\ &= (I - 2k_1\lambda_{min}(H))e^T e + (I^{-1}2k_3\lambda_{min}(H) - 2(a \\ &\quad + y))s^T s + \mathcal{V} \end{aligned}$$

where $\nu = -2k_2\sum_{i=1}^{n}\sum_{j=0}^{n} a_{ij}e_i^T sig(e_i(t) - e_j(t))^\alpha - 2k_4\sum_{i=1}^{n}\sum_{j=0}^{n} a_{ij}s_i^T sig(s_i(t) - s_j(t))^\alpha$.

Note that

$$\begin{aligned} &2\sum_{i=1}^{n}\sum_{j=0}^{n} a_{ij}s_i^T sig(s_i - s_j)^\alpha \\ &= 2\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}s_i^T sig(s_i - s_j)^\alpha \\ &+ 2\sum_{i=1}^{n} a_{i0}s_i^T sig(s_i - s_j)^\alpha \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(s_i - s_j)^T sig(s_i - s_j)^\alpha \\ &+ 2\sum_{i=1}^{n} a_{i0}(s_i)^T sig(s_i)^\alpha \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{m} a_{ij}|s_{il} - s_{jl}|^{\alpha+1} \\ &+ 2\sum_{i=1}^{n}\sum_{l=1}^{m} a_{i0}|s_{il}|^{\alpha+1}. \end{aligned}$$

So, we have

$$\begin{aligned} \mathcal{V} = &-k_2\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{m} a_{ij}|e_{il} - e_{jl}|^{\alpha+1} \\ &- 2k_2\sum_{i=1}^{n}\sum_{l=1}^{m} a_{i0}|e_{il}|^{\alpha+1} \\ &- k_4\sum_{i=1}^{n}\sum_{j=1}^{n}\sum l = 1 a_{ij}|s_{il} - s_{jl}|^{\alpha+1} \\ &- 2k_4\sum_{i=1}^{n}\sum_{l=1}^{m} a_{i0}|s_{il}|^{\alpha+1} \\ &\le -k_2[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{m} a_{ij}|e_{il} - e_{jl}|^2 \\ &+ 2\sum_{i=1}^{n}\sum_{l=1}^{m} a_{i0}|e_{il}|^2]^{\frac{1+\alpha}{2}} \\ &- k_4[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum l = 1 a_{ij}|s_{il} - s_{jl}|^2 \\ &+ 2\sum_{i=1}^{n}\sum l = 1 a_{i0}|s_{il}|^2]^{\frac{1+\alpha}{2}} \\ &\le -k_2[\sum_{i=1}^{n}\sum j = 1 a_{ij}(e_i - e_j)^T(e_i - e_j) \\ &+ 2e^T Be]^{\frac{1+\alpha}{2}} - k_4[\sum i = 1\sum j = 1 a_{ij}(s_i - s_j)^T(s_i \\ &- s_j) + 2s^T Bs]^{\frac{1+\alpha}{2}} \\ &\le -k_2(2e^T(\mathrm{H} \otimes I_m)e)^{\frac{1+\alpha}{2}} - k_4(2s^T(\mathrm{H} \otimes I_m)s)^{\frac{1+\alpha}{2}} \\ &\le -\overline{k}\lambda_{min}(\mathrm{H})[((e^T e)^{\frac{1+\alpha}{2}} + (s^T s)^{\frac{1+\alpha}{2}}] \\ &\le -\overline{k}\lambda_{min}(\mathrm{H})[((e^T e) + (s^T s)]^{\frac{1+\alpha}{2}} \\ &\le -\overline{k}\lambda_{min}(\mathrm{H})V^{\frac{1+\alpha}{2}} \end{aligned}$$

with $B = diag(a_{10}, a_{20}, a_{30}, \cdots, a_{n0})$, $\overline{k} = min\left\{2^{\frac{1+\alpha}{2}}k_2, 2^{\frac{1+\alpha}{2}}k_4\right\}$.

Substituting $\mathcal{V}$ into V_1

$$\begin{aligned}\dot{V} &\le (I-2k_1\lambda_{min}(H))e^Te+(I^{-1}2k_3\lambda_{min}(H)\\&\quad-2(a+y))s^Ts-\bar{k}\lambda_{min}(H)V^{\frac{1+\alpha}{2}}\\&\le -l_1e^Te-v_1s^Ts-\bar{k}\lambda_{min}(H)V^{\frac{1+\alpha}{2}}\\&\le -l_1V-\bar{k}\lambda_{min}(H)V^{\frac{1+\alpha}{2}}. \end{aligned}\tag{29}$$

Step 2: When $t\in T_s$, it can be acquired that

$$\begin{aligned}\dot{V} &= 2e^T\dot{e}+s^T\dot{s}\\&= 2e^T(s+\bar{v})+2s^T(-(a+y)s-\dot{\bar{v}}-y\bar{v})\\&\le \mathscr{I}e^Te+(\mathscr{I}^{-1}-2(a+y))s^Ts \le l_2V. \end{aligned}\tag{30}$$

And $V(t)\equiv 0$, according to Lemma 6, $\forall t>T$, where T is given by

$$T \le T_1+\frac{2}{1-\alpha}\frac{ln(1+\frac{l_1}{\bar{k}}\lambda_{min}(H)V^{\frac{1-\alpha}{2}}(0))}{(l_1-(l_1+l_2)\varepsilon)} \tag{31}$$

So, $V(t)$ converging to 0 shows that the systems (1) have reached synchronization under aperiodically intermittent communication within the time [0, T] by employing the designed protocol (25).

Remark 3.3. From the selection conditions of control gain, it can be seen that its magnitude depends on the number of manipulators in the network and the eigenvalues of the Laplacian matrix. These factors constitute global information that must be precisely known to the designer.

Remark 3.4. If the dynamics equation of the leader is

$$\ddot{x}_0 = f(x_0,\dot{x}_0)S. \tag{32}$$

The control protocol is changed to

$$\begin{aligned}\tau_i(t) &= M_i(x_i)(\psi_i+\dot{\phi}_i+y\phi_i-aS_i+\ddot{x}_0)\\&\quad - M_i(x_i)y(x_i-\hat{v}_i)+C_i(x_i,\dot{x}_i)\dot{x}_i+G_ix_i). \end{aligned}\tag{33}$$

This means that the leader's information is known by other followers. To better reach the control objectives, a distributed controller needs to be designed, which is huge challenge for us to achieve synchronization in finite-time.

Remark 3.5. Our research on the finite-time synchronization of multi-manipulator systems under aperiodically intermittent communication has notable practical applications. In industrial automation, it can enhance the efficiency and flexibility of assembly lines, effectively dealing with communication disruptions caused by electromagnetic interference or network congestion. In disaster response scenarios like earthquake-stricken areas or hazardous chemical leak sites, this approach enables multi-manipulator robotic systems to operate smoothly even with unstable communication, facilitating search and rescue, debris removal, and environmental monitoring tasks.

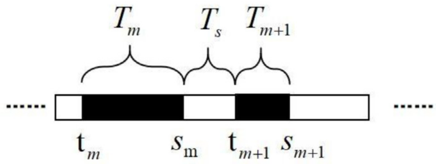

Fig. 1. Diagram of intermittent communication.

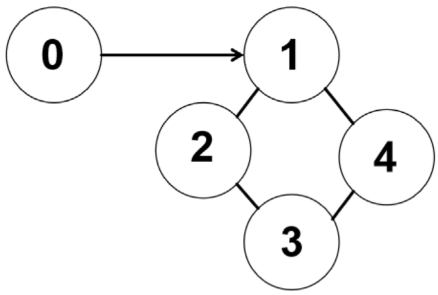

Fig. 2. Topographical graph.

4 Simulation example

This part aims to provide a numerical simulation example to verify our control protocol. The system considered in the simulation has four followers and one leader, and the connection paths between them are shown in Figure 2.

Consider two-link multi-manipulator systems, and the state equation of each manipulator is established by systems (1), where $v_i=[\dot{x}_{i(1)}\dot{x}_{i(2)}]^T$, $\tau_i=[\tau_{i(1)}\tau_{i(2)}]^T$, the inertia matrix $M_i(x_i)\in\mathscr{R}^{2\times 2}$, the Coriolis and centripetal forces matrix $C_i(x_i,\dot{x}_i)\in\mathscr{R}^{2\times 2}$ and the Gravity vector $G_i(x_i)\in\mathscr{R}^2$ are given by the following equation

$$M_i(q_i)=\begin{bmatrix} a_1+a_2+4a_3cos(q_{i(2)}) & a_2+2a_3cos(q_{i(2)})\\ a_2+2a_3cos(q_{i(2)}) & a_2\end{bmatrix} \tag{34}$$

$$C_i(q_i\dot{q}_i)=\begin{bmatrix} -2a_3\dot{q}_{i(2)}\,sin(q_{i(2)})-a_3\dot{q}_{i(2)}sin(q_{i(2)})\\ -a_3\dot{q}_{i(2)}sin(q_{i(2)}) \qquad\qquad 0\end{bmatrix} \tag{35}$$

$$G_i(q_i)=\begin{bmatrix} a_4gsin(q_{i(1)})\\ a_5gsin(q_{i(2)})cos(q_{i(2)})\end{bmatrix} \tag{36}$$

where $a_1=2.25\,kg\times m^2$, $a_2=0.1\,\text{kg}\times\text{m}^2$, $a_3=0.04\,\text{kg}\times\text{m}^2$, $a_4=0.2\,\text{kg}\times\text{m}^2$, $a_5=1\,\text{kg}\times\text{m}^2$, $g=9.8\,\text{m}\times\text{s}^2$. The initial position information of following agents are given as

$$x_1(0)=\begin{bmatrix}0.9\\0.68\end{bmatrix}(rad), x_2(0)=\begin{bmatrix}2.5\\2.26\end{bmatrix}(rad), x_3(0)=\begin{bmatrix}1.8\\1.16\end{bmatrix}(rad), x_4(0)=\begin{bmatrix}0.56\\0.36\end{bmatrix}(rad).$$

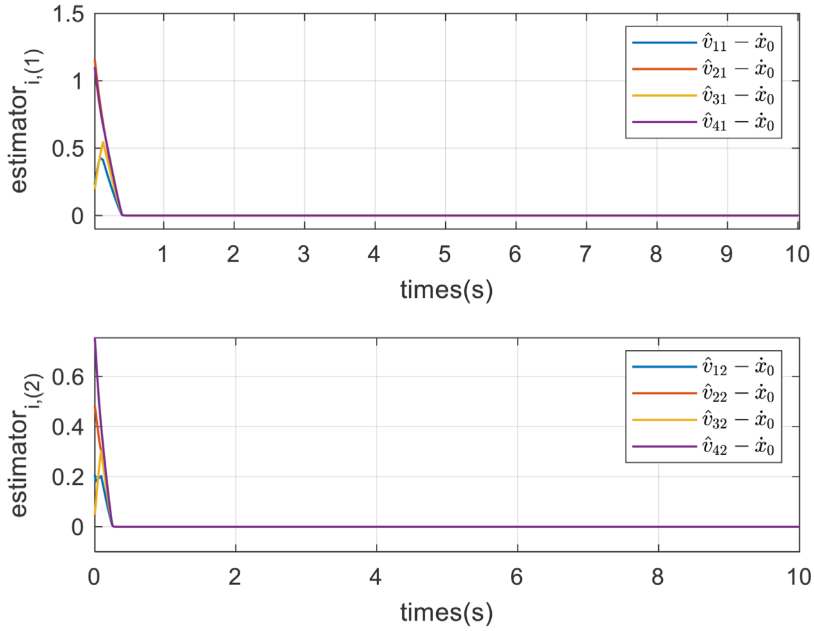

Fig. 3. Error between the speed of the estimator $\hat{v}_{i(1)}, \hat{v}_{i(2)}, \hat{v}_{i(3)}, \hat{v}_{i(4)}$and the speed of the leader $\dot{x}_0$.

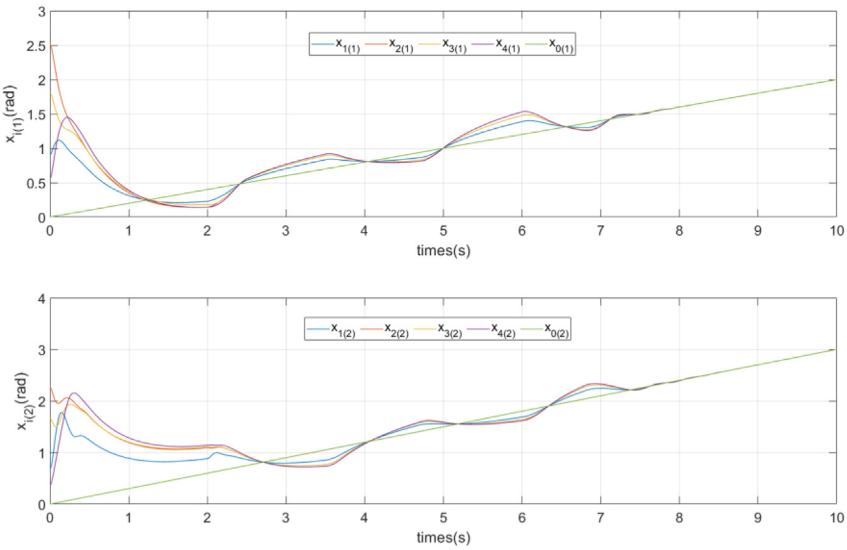

Fig. 4. Position of agentsx_1, x_2, x_3, x_4, x_0.

The initial velocity information of following agents are given as

$$v_1(0)=\begin{bmatrix}0.63\\0.98\end{bmatrix}(rad/s),v_2(0)=\begin{bmatrix}0.63\\0.6\end{bmatrix}(rad/s),v_3(0)=\begin{bmatrix}0.91\\0.57\end{bmatrix}(rad/s),v_4(0)=\begin{bmatrix}0.32\\0.56\end{bmatrix}(rad/s).$$

The position and velocity of leader agent are given as

$$x_0(t) = \begin{bmatrix}0.2t\\0.3t\end{bmatrix}(rad), v_0(t) = \dot{x}_0\,(t) = \begin{bmatrix}0.2\\0.3\end{bmatrix}(rad/s).$$

Suppose that two-link multi-manipulator systems are moving with a leader as the desired target. Then, according to the algorithm, the following parameters will be obtained. Consider $k_1=8$, $k_2=3$, $k_3=3$, $k_4=2$, $a=0.5$, $y=1.2$, $u=0.7$, $\alpha=0.3$. Based on the above parameters, the values for l_1 and l_2 are 2.1, 1.6,then $\varepsilon=0.567$. We set the interrupt communication to [0.5, 2]∪[2.5, 3.5]∪[3.7, 4.7]∪[5,6]∪[6.3, 6.8].

From Figure 3, it can be seen that the designed estimator can estimate the system's velocity in finite-time. Figures 4 and 5 show the position and velocity trends of the leader and the follower under the designed synchronization protocol, demonstrating gradual convergence to the leader.

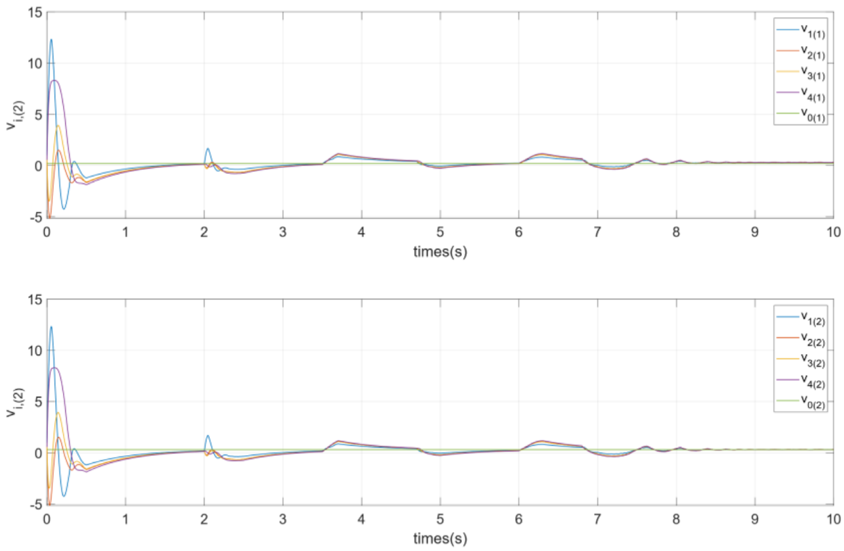

Fig. 5. Velocity of agentsv_1, v_2, v_3, v_4, v_0.

Finite-time convergence of the multi-manipulator system is guaranteed by the designed control protocol despite intermittent communication, as demonstrated in the results.

5 Conclusion

This paper has investigated the finite-time synchronization of multi-manipulator systems under aperiodically intermittent communication. A velocity estimator and a finite-time controller have been designed by constructing auxiliary variables. Then, the designed control algorithm has proven that the synchronization can be reached in finite-time. The theoretical proof of our control algorithm confirms that finite-time synchronization can be achieved, marking a major leap forward in the research of multi-manipulator systems with irregular communication. Finally, simulation results have been used to validate all the theories in this paper. The consideration of aperiodically intermittent communication enriches the theoretical system of multi-manipulator control and provides an essential basis for the design and improvement of robotic systems in scenarios with unstable or limited communication, including remote control operations, mobile robotic tasks, and distributed industrial automation processes. In the future, we will continue to study the fixed control and energy estimation of multi-manipulator systems under aperiodic intermittent control.

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Conflicts of interest

The authors have nothing to disclose.

Data availability statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Author contribution statement

Conceptualization, Hui Shen and Dan Liu; Methodology, Hui Shen; Software, Hui Shen; Validation, Dan Liu; Formal Analysis, Dan Liu; Investigation, Hui Shen and Dan Liu; Resources, Binrui Wang; Data Curation, Hui Shen; Writing – Original Draft Preparation, Hui Shen; Writing – Review & Editing, Dan Liu; Visualization, Dan Liu; Supervision, Binrui Wang; Project Administration, Binrui Wang; Funding Acquisition, Binrui Wang.

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