Air conditioning reliability analysis based on dynamic Bayesian network and Markov model

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Abstract. With the popularization of the air conditioning, its reliability during operation has gradually become a focus of attention. However, due to the uncertainty in the reliability analysis process, the accuracy of the results will be affected. To overcome this challenge, a method for air conditioner reliability analysis combining Dynamic Bayesian Network (DBN) and Markov Model (MM) is proposed. Firstly, orthogonal defect classification (ODC) is used to statistic and analyze the defect data of the air conditioning system, and the network structure of the DBN is determined based on the results of the analysis. Then, the state transfer probability of each node is obtained by MM, and then the reliability, steady state availability, and maintainability of the air conditioning system are analyzed. Finally, the effectiveness of the method is verified by a case study of air conditioning failure data. The results show that the steady state availability of the air conditioning system in this case is 0.996.

Keywords: Air conditioning system / dynamic Bayesian network / Markov model / reliability analysis

1 Introduction

As a temperature regulator, air conditioning has been widely used in various residential and commercial buildings. According to statistics, the average number of air conditioners owned by every 100 Chinese households in 2021 is 131.2 units, an increase of 11.5% over 2020 [1]. However, when the components of the air conditioning system fail or degrade, not only its comfort will decrease, but may also lead to a series of safety accidents. Therefore, it is necessary to perform a reliability analysis of household air conditioning systems.

In recent years, the reliability of the air conditioning system has received more and more attention. Du et al. [2] had combined cluster analysis and principal component analysis to achieve reliability evaluation of sensors in refrigeration and air conditioning systems. In order to predict and evaluate the ventilation system quickly and accurately, Liu et al. [3] proposed an improved RNG k-ε model, and verified the reliability of the model through simulation. Chang et al. [4] adopted failure mode and effect analysis (FMEA) to analyze the failure of scroll compressor for system air conditioning, and evaluated the reliability of scroll compressor through design of experiments (DOE). Mostafa et al. [5] studied the reliability of the vapor compression refrigeration system (VCRS) components and optimized the overall reliability of the system by using the reliability block diagram (RBD) method and single-objective genetic algorithm (GA). Unfortunately, the incompleteness and complexity of system information can lead to many uncertainties in the analysis process, which traditional reliability analysis methods are difficult to deal with.

Bayesian Network (BN) is a probabilistic graph model that can effectively solve various uncertainty problems [6]. It has been introduced into the reliability analysis of complex systems and has achieved successful applications, such as subsea production systems [7–9], electronic systems [10,11], manufacturing systems [12–14], computer numerical control machine tools [15,16], wind turbines [17,18]. However, the traditional BN can only carry out static analysis, and it is difficult to model the transition process among multiple states. The Markov model (MM) is a statistical method, which allows simulating the stochastic behaviour of the system in continuous or discontinuous change states with respect to time or space [19]. Therefore, researchers integrate traditional BN with MM to form a Dynamic Bayesian Network (DBN) to solve the problem of component degradation over time. In order to dynamically predict the impact of the microbiological corrosion on the residual strength and survival likelihood of the pipeline, a dynamic safety assessment method combining BN and MM was proposed by Adumene et al. [20]. Wang et al. [21] used
DBN to predict the dynamic probability of offshore platform fire and determined the main factors leading to fire incidents through sensitivity analysis. Jiang et al. [22] performed reliability and sensitivity analysis of the kick detection sensor networks based on DBN, and realized the optimal logical combination of sensors according to the analysis results. Li et al. [23] proposed a reliability analysis method of multi-state elements based on DBN and applied it to control units to determine the reliability values of control units under different modes.

In this study, a DBN model is proposed to analyze the reliability of household air conditioning system by combining DBN and MM. We apply the model application to analyze the reliability and maintainability of household air conditioners in a medium-sized city in China. The reliability, the steady state availability and the maintainability of household air conditioners are evaluated.

2 Methods

2.1 Dynamic Bayesian network

When adopting the DBN model to analyze the reliability of household air conditioning systems, the following assumptions are made to simplify the modelling process due to the complexity of the probability distribution of random variables [24]:

- Assume that the network structure and the conditional probabilities of the nodes are time-invariant.
- Assume that the probability of future moment is only related to the current moment, but not to the past. The mathematical expression is as follows [25]:

\[
P(X_{t+1}|X_1, X_2, \ldots, X_t) = P(X_{t+1}|X_t).\]  

(1)

Based on the above assumptions, the DBN can be defined as \((B_0, B_\infty)\), where \(B_0\) is the prior network, which represents the probability distribution of the initial state, and \(B_\infty\) is the transition network, representing the state transition probability between nodes of two adjacent time slices. The DBN model diagram is shown in Figure 1.

2.2 DBN construction

DBN construction can usually be divided into two major steps: network structure construction and node conditional probability determination [26]. When constructing the network structure, it is necessary to firstly identify the variables and select the network nodes, and then determine the network structure based on the relationships between the nodes. The identification of variables is usually based on expert knowledge. However, it is a significant challenge for the experts since failures in air conditioning systems are caused by a variety of factors. Therefore, the orthogonal defect classification (ODC) is introduced for statistical analysis of household air conditioning defect data.

ODC is a defect classification method developed by IBM [27]. The method is a multi-dimensional analysis of defects by extracting and quantifying the key attributes in defects that are orthogonal to each other, so that each defect can be uniquely classified into a class in each dimension, which can effectively avoid the problem of classification ambiguity and inaccuracy. The three key orthogonal defect attributes of events, sources and objects were extracted and the network structure of the DBN model was constructed by statistics and analysis of the air conditioning defect data. In this paper, we combined expert knowledge and the collected defective data to determine the conditional probability of each node, so as to reduce the influence of subjective factors.

2.3 DBN parameters

The DBN parameter model consists of intra time slice parameter model and inter time slice parameter model. The intra time slice parameter model is the conditional probability of each node. There are two common methods to determine the conditional probabilities, one is based on the empirical knowledge of experts, and the other is parameter learning by collecting a large amount of data. The former is simple and convenient, but highly subjective. The latter is objective, but it is difficult to collect a full set of data.

The inter time slice parameter model is the transfer probability between time slices. Since the dynamic probabilistic process conforms to the Markov property, a two-state Markov model is introduced to determine the state transfer probabilities. The state transition diagram of the two-state Markov model is shown in Figure 2, where \(\lambda_{0,1}, \lambda_{1,0}, \lambda_{0,0}, \lambda_{1,1}\) denotes the transfer probability of node states from 0 to 1, 1 to 0, 0 to 0, 1 to 1 at time change \(\Delta t\), respectively.

According to the state transition diagram in Figure 2, the Markov transition matrix of the household air conditioning system can be obtained:

\[
Q = \begin{bmatrix}
\lambda_{0,0} & \lambda_{0,1} \\
\lambda_{1,0} & \lambda_{1,1}
\end{bmatrix}.
\]  

(2)
Assuming that the failure time and repair time of the air conditioning system follow exponential distribution, the state transition probability in matrix $Q$ can be determined by the following equation:

\[
\lambda_{0,0} = e^{-\lambda \Delta t},
\]

\[
\lambda_{0,0} = e^{-\lambda \Delta t},
\]

\[
\lambda_{1,1} = 1 - e^{-\mu \Delta t},
\]

\[
\lambda_{1,0} = 1 - e^{-\lambda \Delta t},
\]

where $\lambda$ is the failure rate, $\mu$ is the repair rate, and the values of $\lambda$ and $\mu$ are obtained from references [28].

### 2.4 Reliability and availability evaluation

In practical engineering, household air conditioning systems are repairable. For repairable systems, the availability is an important indicator to measure the reliability of the system.

The probability distribution vector of each state of the system at time $t$ of the household air conditioning system is defined as:

\[
P(t) = [P_0(t), \ldots, P_i(t)],
\]

\[
P(t)Q = P'(t),
\]

where $P_i(t)$ is the probability that the household air conditioning system is in state $i$ at time $t$, which can be deduced from the initial state by equation (2)–(8). $P'(t)$ is the time derivative of $P(t)$.

Since the household air conditioning system can only operate normally in state 0, the transient availability of the system can be expressed as [29]:

\[
A_0(t) = P_0(t),
\]

When $t \rightarrow \infty$, the transient availability of each state is transformed into steady state availability, and the probability distribution vector of each state can be expressed as:

\[
P = [P_0, \ldots, P_1],
\]

where $P_i$ is the probability that the household air conditioning system is in state $i$ at stability.

The Kolmogorov forward equation of the steady state can be expressed as:

\[
\begin{aligned}
P_0Q &= 0 \\
\sum_{i=0}^{1}P_i &= 1
\end{aligned}
\]

Therefore, the steady state availability $A_0$ of the household air conditioning system can be obtained as:

\[
A_0 = P_0.
\]

### 3 Case study

#### 3.1 DBN model for household air conditioning

A total of 1172 households’ air conditioning system failure information was collected by the air conditioning failure statistics analysis software. Based on the three attributes of event, source and object, the obtained defect information is statistically analyzed, and the statistical diagram of defect information of household air conditioners is shown in Figure 3.

The structure of DBN model of household air conditioning system is constructed according to the results of statistical analysis as shown in Figure 4, where T represent household air conditioning system, S1–S5 represent refrigeration system, electric control system, ventilation system, box system, accessories respectively.

After completing the structure construction of DBN model for household air conditioning system, the network parameters need to be determined. The prior probabilities of nodes S1–S5 can be obtained by the household air conditioning failure condition node and repair rate $\mu$ can be obtained from references, as shown in Table 1. The expert empirical knowledge and the collected data are used to obtain the conditional probabilities of the nodes. The failure rate $\lambda$ and repair rate $\mu$ are obtained from references, as shown in Table 2, and then the conversion probability is calculated according to equation (2)–(6). Based on the obtained network structure and parameters, GeNe software was used to construct the simulation figure of DBN model for household air conditioning system, as shown in Figure 5.

#### 3.2 Evaluation results and discussion

The reliability and maintainability of each subsystem and the whole household air conditioning system were obtained by conducting reliability analysis of the household conditioning system, as shown in Figure 6 and 7.
Fig. 3. Statistical diagram of ODC defect information. (a) The statistical data of event. (b) The statistical data of event. (c) The statistical data of object.

Fig. 4. The structure of DBN model of household air conditioning system.

Table 1. The prior probabilities of nodes S1–S5.

<table>
<thead>
<tr>
<th>Node</th>
<th>Prior probabilities</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.5629</td>
<td>0.4371</td>
</tr>
<tr>
<td>S2</td>
<td>0.9285</td>
<td>0.0715</td>
</tr>
<tr>
<td>S3</td>
<td>0.9671</td>
<td>0.0329</td>
</tr>
<tr>
<td>S4</td>
<td>0.8376</td>
<td>0.1624</td>
</tr>
<tr>
<td>S5</td>
<td>0.8701</td>
<td>0.1299</td>
</tr>
</tbody>
</table>

Table 2. Failure rate $\lambda$ and repair rate $\mu$.

<table>
<thead>
<tr>
<th>Node</th>
<th>Failure rate $\lambda$</th>
<th>Repair rate $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>$4.25\times10^{-5}$</td>
<td>0.1233</td>
</tr>
<tr>
<td>S2</td>
<td>$5.29\times10^{-5}$</td>
<td>0.0471</td>
</tr>
<tr>
<td>S3</td>
<td>$6.18\times10^{-6}$</td>
<td>0.0503</td>
</tr>
<tr>
<td>S4</td>
<td>$3.11\times10^{-7}$</td>
<td>0.1429</td>
</tr>
<tr>
<td>S5</td>
<td>$4.61\times10^{-7}$</td>
<td>0.0552</td>
</tr>
</tbody>
</table>
It can be clearly seen from Figure 6 that if the maintenance factor is not taken into consideration, the reliability of the whole system and all the subsystems will decrease with time and gradually tend to 0. After 50,000 hours of operation, although the reliability of the box system and auxiliary components is still maintained at more than 90%, the reliability of the whole system and the electronic control system is close to 0, which means that the household air conditioning and the electronic control system are a faulty state. The reason for this is that the electronic control system contains a lot of electronic components, and the failure of just one component may lead to problems in the corresponding circuit. This shows that the electronic control system has the greatest impact on the whole system and is the weak part of the air conditioning system. And this is the same as the statistical result of ODC, which proves the validity of the model.

As can be seen in Figure 7, the maintainability of the whole system and the subsystems gradually increases to 1 over time. The refrigeration system and the box system return to their optimum condition after 82 hours of maintenance. The reason for this is that the components of the two subsystems have simpler structures and could be easily maintained. However, the electronic control system, which is the main system to realize the functions of the household air conditioning, needs 188 hours of maintenance to be restored to the optimum condition. It is because the electronic control system contains specific functional circuits with complicated structure, which makes the maintenance need to spend more time. And the whole system is restored to the optimal condition after the electronic control system is restored to the optimal condition. This demonstrates that the maintainability of the whole system is more closely related to the electronic control system.

Through equations (7)–(12) and Table 2, the transient availability and steady state availability of the household air conditioning system can be obtained, as shown in Figure 8. From the figure, it can be seen that if the maintenance factor is considered, the availability of the household air conditioning system decreases rapidly to about 0.996 in the first 150 hours. After a further period of about 50 hours, the availability of the system gradually becomes stabilized. And the steady state availability of the household air conditioning system can be obtained as 0.996, which indicates that the safety of the household air conditioning system in the area is good.

By comparing Figure 6 and Figure 8, it can be seen that when considering the maintenance factor, the availability of the household air conditioning system has a significant increased, which indicates that the maintenance factor has a greater impact on the operational reliability of household air conditioning. As a result, maintenance is an effective means to guarantee the operation safety and reliability of household air conditioners, which can detect and deal with potential hazards timely and reduce the failure of the
system or components, so as to avoid the occurrence of safety accidents. According to the results of the evaluation, the maintenance interval for household air conditioning systems is recommended to be 188 hours.

4 Conclusions

In this paper, a reliability analysis method based on DBN and Markov model for household air conditioning systems is proposed. The orthogonal defect classification is introduced into the structure construction of DBN, and the three key attributes of event, source and object are extracted by statistical analysis of 1172 air conditioner defect data, and the DBN model is constructed based on expert knowledge and statistical analysis results. Then, the reliability of the household air conditioning system is analyzed by determining the state transfer probability of the nodes using a Markov model. The results show that the whole household air conditioning system will fail after 50,000h without considering maintenance. If the maintenance process is considered, the system steady state availability $A_0$ is determined as 0.996. On the other hand, by analyzing the maintenance function of each subsystem, the optimal maintenance cycle of the household air conditioning system is determined to be 188h. Therefore, this study is useful for improving the reliability of household air conditioners, reducing the waste of resources, and can facilitate the work of maintenance personnel through the obtained maintenance intervals.

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Conflict of Interest
We declare that there is no conflict of interest in this paper.

Data availability statement
The data that have been used are confidential.

Author contribution statement
Writing-original draft: Jiaqi Xu, Qiang Wang. writing-review and editing: Juan Zhou, Linlin Wu, Jiayan Chen, and Haiting Zhou.

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