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Estimation of parallelism measurement uncertainty according to the Geometrical Product Specifications standard using coordinate measuring machine

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Abstract. The dimensional and geometrical specifications control of a mechanical part is commonly done using a coordinate measuring machine. Collecting and processing measurement data through a probe, it allows measuring the conformity of the part according to specific tolerances. An accurate estimation of the uncertainty of measurement is critical; it is within this framework that our study is oriented; we aim to estimate the measurement uncertainty of parallelism error using the Guide to the Expression of Measurement Uncertainty (GUM), then we will proceed to a Monte Carlo simulation to compare the obtained uncertainty, we will then proceed to an inter-laboratory comparison to validate our model. Our contribution is based on a more detailed and precise estimation of the uncertainties of the measurement process taking into accounts the calibration of the machine and the propagation of uncertainties.

Keywords: Coordinate measuring machine (CMM) / parallelism error / guide to the expression of uncertainty in measurement (GUM) / Monte Carlo simulation (MCS) / inter-laboratory comparison (ILC) / parallelism uncertainty

1 Introduction

Considering the increasingly demanding quality that the modern industry is experiencing, accurate and precise inspection tools are required to go along the technological progress of manufactured parts, thus making the coordinate measuring machine (CMM) one of the widely used solution to control dimensional and geometrical specifications. Collecting inspected points through a probe, the software then processes data and creates fitted surfaces using the least square method, then measures the conformity of the part according to a specific criterion, which is a parallelism error in our case. This succession of steps is subject to a propagation of uncertainties, which if not estimated right, can lead to a high risk on the declaration of conformity. An accurate estimation of the uncertainty of measurement is critical and has a direct impact on decision-making; it is within this framework that our study is oriented.

Several studies have been made in order to estimate the form error measurement uncertainty in a CMM. Rosenda et al. [1] proposed a simplified model to estimate the measurement uncertainty deviations in CMM when determining circularity and cylindricity based on identifying the variables that influence the determination of geometry deviation. Ilyas Khan et al. [2] coded a genetic algorithm based on the minimum zone criterion. The algorithm estimates the parallelism error considering both datum and measured planes flatness default. Zha et al. [3] propose a strategy to estimate and minimize parallelism default using CMM. Wladyslaw and Wojciech developed in reference [4] a CMM measurement uncertainty evaluation software fully consistent with the GPS norm. Wojciech [5] then proposed different models of geometrical deviations associated uncertainties, including parallelism.

Referring to the GUM approach and the Monte Carlo method is usually done to estimate the measurement uncertainty. Egidi et al. [6] developed a model using the Monte Carlo method to estimate the uncertainty during the measurement of roundness on a coordinate measuring machine. Balasubramanian et al. [7] estimated uncertainty in angle measurement using the GUM method taking into consideration the geometrical errors, temperature, vibrations, and probing strategy. Using a comparison between these two methods (GUM and the MCM) to validate an uncertainty measurement model, has been proven to give consistent results, it's within this framework that Jalid

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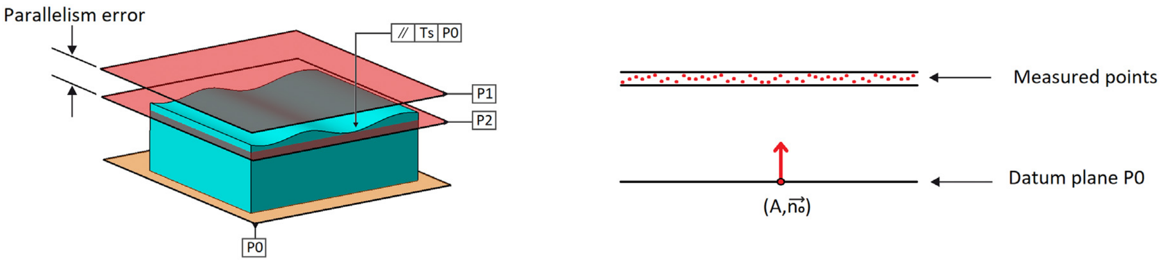

Fig. 1. Parallelism error.

proposed a comparison of these two methods on the estimation of flatness uncertainty [8] based on the orthogonal distance regression (ODR) that provided the parameters of the substitute plane and their uncertainties, then studied the influence of sample size on the flatness estimation and uncertainty [9].

In this paper, we aim to introduce a parallelism error model and estimate the associated measurement uncertainty using the guide to the expression of measurement uncertainty [10] and compare it to the results obtained with the Monte Carlo method [11], and then we will proceed to an interlaboratory comparison to validate our model. Our model combines the experimental and the analytical methods to estimate the parallelism error and the associated uncertainty taking into consideration the calibration of the machine and the propagation of uncertainties through the measurement process and following the normative guidelines.

2 Materials and methods

In order to estimate the parallelism associated uncertainty, we applied the following approach:

- Setting a parallelism error equation according to the GPS standard.
- Applying the GUM uncertainty propagation model to the parallelism equation.
- Coding an algorithm on Matlab to estimate the datum plane parameters and associated variance-covariance matrix.
- Estimating the variance-covariance matrix of the measured points $p_{\max}$ and $p_{\min.}$
- Calculation the parallelism uncertainty using the GUM
- Validating the GUM results using a Monte Carlo simulation.
- Confirmation of our model through an inter-laboratory comparison using normalized error.

2.1 Parallelism error mathematical model

Based on ISO 1101 [12], Parallelism is an orientation tolerance, defined in as the minimum distance between two theoretical parallel planes P1 and P2, both parallel to the datum plane P0, within which all measured points lie inside (Fig. 1).

Let $p_{\max}$ and $p_{\min}$ be the two most distant measured points such as:

$p_{\max}\,(x_{\max}, y_{\max}, z_{\max}) \in P1$ and $p_{\min}\,(x_{\min}, y_{\min}, z_{\min}) \in P2$

The parallelism can be expressed in the following way:

$$\begin{aligned} \mathrm{dp} &= |\overrightarrow{p_{\max}p_{\min}}.\overrightarrow{n_0}| \\ &= \left| \begin{pmatrix} x_{p_{\min}} - x_{p_{\max}} \\ y_{p_{\min}} - y_{p_{\max}} \\ z_{p_{\min}} - z_{p_{\max}} \end{pmatrix} \begin{pmatrix} n_{xo} \\ n_{yo} \\ n_{zo} \end{pmatrix} \right| \\ &= |(x_{p_{\min}} - x_{p_{\max}})n_{xo} + \left(y_{p_{\min}} - y_{p_{\max}}\right)n_{yo} \\ &\quad + (z_{p_{\min}} - z_{p_{\max}})n_{zo}| \end{aligned} \tag{1}$$

where $\overrightarrow{n_0}$ $(n_{xo},\ n_{yo},\ n_{zo})$ represents the datum plane's normal vector.

2.2 GUM uncertainty propagation model

The law of propagation of uncertainty GUM (Guide to the expression of uncertainty in measurement) is an analytic method based on a first order Taylor expansion of a function $Y = f(X_1, X_2, X_3, \ldots, X_n)$. The final uncertainty is obtained by propagating the elementary components through a linear approximation. To estimate the parallelism error uncertainty modeled in the equation (1), we relied on the variance propagation law given by the GUM guide applied to $\mathrm{dp} = f\left(x_{p_{\min}}, y_{p_{\min}}, z_{p_{\min}}, x_{p_{\max}}, y_{p_{\max}}, z_{p_{\max}}, n_{xo}, n_{yo}, n_{zo}\right)$:

$$u_c^2(\mathrm{dp}) = \sum_{i=1}^{N}\left[\left(\frac{\partial dp}{\partial X_i}\right)\right]^2 var(X_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\frac{\partial dp}{\partial X_i}\right)\left(\frac{\partial dp}{\partial X_j}\right)$$
$$cov(X_i, X_j) = \sum_{i=1}^{N}\sum_{j=1}^{N}\left(\frac{\partial dp}{\partial X_i}\right)\left(\frac{\partial dp}{\partial X_j}\right)cov(X_i, X_j) = JMJ^T, \tag{2}$$

where J is the Jacobian matrix given bellow:

$$J = \begin{bmatrix} \frac{\partial dp}{\partial n_x} & \frac{\partial dp}{\partial n_y} & \frac{\partial dp}{\partial n_z} & \frac{\partial dp}{\partial x_{p_{\min}}} & \frac{\partial dp}{\partial y_{p_{\min}}} \\ \frac{\partial dp}{\partial z_{p_{\min}}} & \frac{\partial dp}{\partial x_{p_{\max}}} & \frac{\partial dp}{\partial y_{p_{\max}}} & \frac{\partial dp}{\partial z_{p_{\max}}} & \end{bmatrix}. \tag{3}$$

M is the variance covariance matrix of the variables $\vec{n}, p_{\min}, p_{\max}$ given as follows:

$$M = \begin{pmatrix} [\vec{n}] & [0] & [0] \\ [0] & [p_{\min}] & [0] \\ [0] & [0] & [p_{\max}] \end{pmatrix}, \tag{4}$$

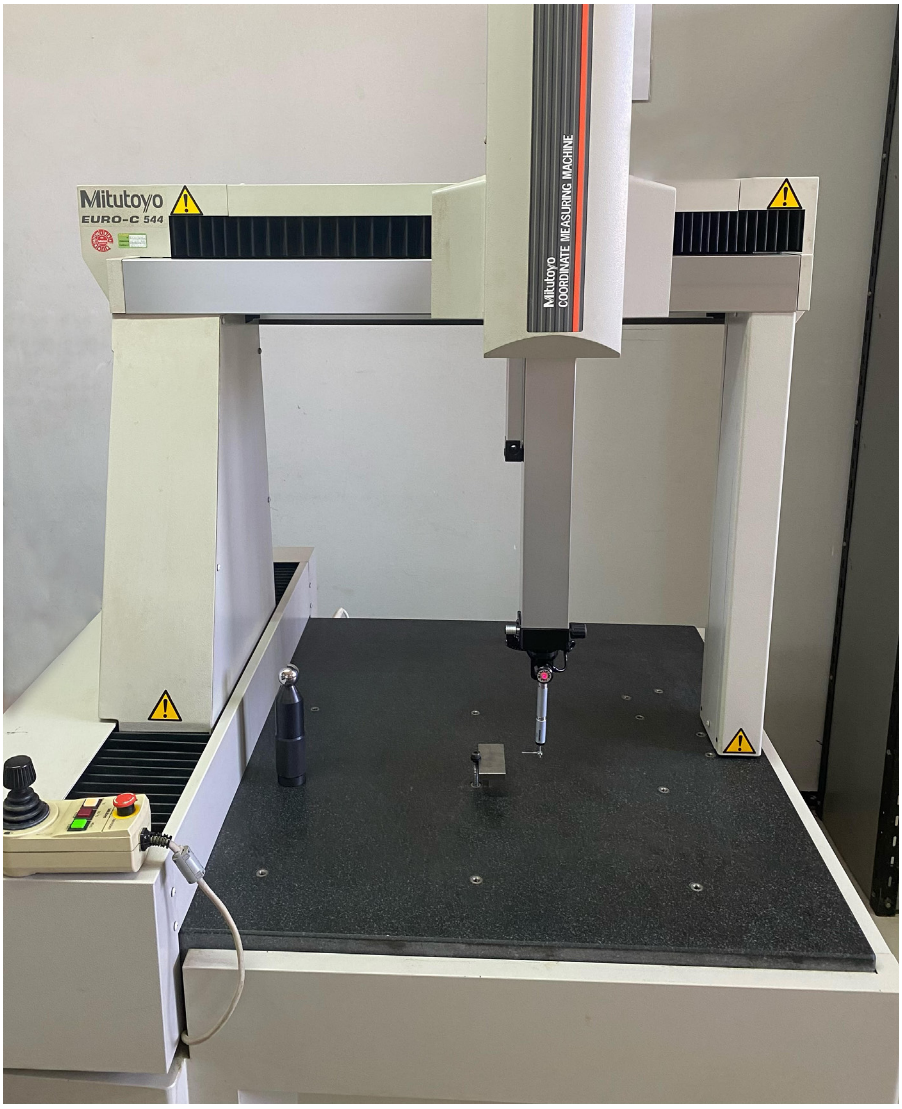

Fig. 2. CMM used.

where the terms of $[\vec{n}]$ are the followings:

$$[\vec{n}] = \begin{pmatrix} var(n_x) & cov(n_x, n_y) & cov(n_x, n_z) \\ cov(n_y, n_x) & var(n_y) & cov(n_y, n_z) \\ cov(n_z, n_x) & cov(n_z, n_y) & var(n_z) \end{pmatrix}, \quad (5)$$

And the terms of $[p_i]$ are the followings:

$$[p_i] = \begin{pmatrix} u_x & u_{xy} & u_{xz} \\ u_{yx} & u_y & u_{yz} \\ u_{zx} & u_{zy} & u_z \end{pmatrix}. \quad (6)$$

2.3 Datum plane parameters ($\vec{n}_0$, A_0) and associated variance-covariance matrix $[\vec{n}]$

Surface fitting is critical step; in our case, it is about associating an ideal plane from the measured points that represents the best the probed plane. To do so, several criteria are available and can be considered according to the nature of the need, the least squares method remains the most frequently used, it is a standard regression approach to approximate the solution of over determined systems:

$$\min \sum_{i=1}^{N} e_i^2.$$

The best fit in the least-squares sense minimizes the sum of squared residuals

$$e_i = A\vec{P}_i.\vec{n},$$

where A and $\vec{n}$ are the substitute plane parameters respectively representing a point belonging to the substitute plane and its normal vector and P_i are the probed

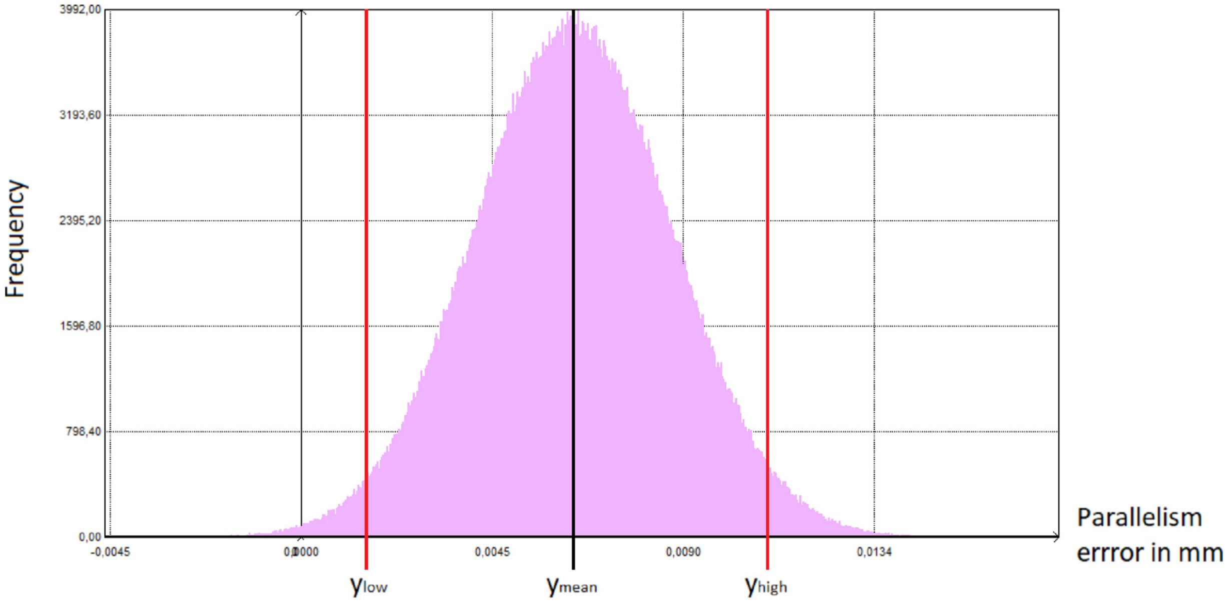

Fig. 3. Distribution law of MCM.

points. This leads us to the equation to minimize as follows:

$$e_i^2 = \left[\begin{pmatrix} x_{p_i} - x_A \\ y_{p_i} - y_A \\ z_{p_i} - z_A \end{pmatrix}\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}\right]^2 .$$
$$df = \left[(x_{p_i} - x_A)n_x + \left(y_{p_i} - y_A\right)n_y + (z_{p_i} - z_A)n_z\right]^2 \quad (7)$$

The starting parameters are critical and can lead to local solution, hence our choice of:

$A_0\left(\frac{\sum x_{p_i}}{n}, \frac{\sum y_{p_i}}{n}, \frac{\sum z_{p_i}}{n}\right)$ the barycenter of probed points.

$\vec{n_0} = \frac{\vec{BC} \wedge \vec{BD}}{|\vec{BC} \wedge \vec{BD}|}$ an initial normal vector based on the most distant points apart B, C and D.

To solve this equation, we have developed an algorithm on Matlab using the function "nlinfit" which allows us to have the parameters of the fitted plane and its associated variance-covariance matrix.

2.4 Estimation of the variance-covariance matrix of a measured point $[p_i]$

CMMs are reliable and accurate. However, several factors may be sources of measurement uncertainty including geometric errors, measuring probe errors, thermally induced errors, vibrations, etc. The measurement uncertainties are not constant over the entire measurement space and varies according to the position of the axis, these same errors are dependent.

Several estimations of the measurement uncertainty have been made, Bahassou et al. [13,14] proposed the following matrix based on the uncertainty propagation law of the CMM calibration model according to the standard ISO 10360 [15]:

$$[p_i] = \begin{pmatrix} u_x & u_{xy} & u_{xz} \\ u_{yx} & u_y & u_{yz} \\ u_{zx} & u_{zy} & u_z \end{pmatrix}.$$

The uncertainties defined in the matrix represent the standard uncertainty, which will be divided by the coverage factor k; we will take $k = 2$ for the rest of the calculations.

2.5 Validation of the GUM model using Monte Carlo method

The estimation of measurement uncertainties by the Monte Carlo method is based on the propagation of distributions of random input quantities through a mathematical model of the measurement model. This is a great alternative when the law of uncertainty propagation (GUM propagation of variances) presents some difficulties, such as when the probability density function for the output variable deviates significantly from a Gaussian distribution giving unrealistic confidence intervals, or when the propagation based on the Taylor expansion to the first order presents an inadequate linearization of the model.

The Monte Carlo approach can also be used as a validation method of the GUM propagation of variances results by following this procedure:

– Applying the GUM uncertainty propagation to calculate $y_{low_{GUM}}$ and $y_{high_{GUM}}$ such as:

$$\begin{cases} y_{low_{GUM}} = y - U(\mathrm{d}p) \\ y_{high_{GUM}} = y + U(\mathrm{d}p) \end{cases}.$$

where $U(\mathrm{d}p)$ represents the uncertainty associated to “dp” obtained by applying GUM

– Applying the Monte Carlo method and deduce $y_{low_{MCS}}$ and $y_{high_{MCS}}$ as they represent the limits for a 95.45% confidence interval ($\mathrm{d}p \pm 2\sigma$) of the generated distribution with known mean value and deviation.

– Setting the tolerance associated with the uncertainty:

$$\zeta = 0.5 \times 10^{r}$$

– Calculating d_{low} and d_{high} given as follows:

$$\begin{cases} d_{low} = |y_{low_{GUM}} - y_{low_{MCS}}| \\ d_{high} = |y_{high_{GUM}} - y_{high_{MCS}}| \end{cases}.$$

– Comparing d_{low} and d_{high} to ζ, the validation criteria is based on comparing the absolute differences of the respective endpoints of the two coverage intervals:

$$\zeta \geq \max(d_{low},\ d_{high}).$$

Then, if both $d_{low}$and d_{high} are no larger than ζ, the comparison is favorable, and the GUM uncertainty framework has been validated in this instance.

3 Results and discussion

The goal of this experimental study is to put into practice the theoretical model made above. To do so, we worked on a prismatic mechanical part, we measured the datum plane for fifteen repetitions in 10 points, to be able to evaluate experimentally the uncertainty matrix of the normal vector and compare it to the one obtained using the “nlinfit” function in *Matlab*, we then measured the substitute surface in 10 points. This method allows to evaluate the effect of the association criterion according to the method of least squares relative to the datum plane.

Laboratory: PCMT.

CMM used: Mitutoyo Euro-C 544.

Software: Geopak.

Measuring volume: $X = 500$ mm, $Y = 500$ mm, $Z = 400$ mm.

Probe type: TP2.

Stylus diameter: 2 mm.

MPE: ET = ±(4 μm + L/200) with L in mm.

Temp: 20 ± 2°C.

The results are as follows:

The datum plane normal vector with its associated uncertainty matrix:

$$\vec{n_0} = \begin{pmatrix} 0.000101811 \\ -0.00010666 \\ 0.99999999 \end{pmatrix} \mathrm{mm}.$$

$$[\vec{n}] = \begin{pmatrix} 1.666E-09 & -1.425E-09 & -5.071E-14 \\ -1.425E-09 & 3.6E-09 & 8.542E-14 \\ -5.071E-14 & 8.542E-14 & 5.067E-18 \end{pmatrix} \mathrm{mm}.$$

The tolerance plane two extreme points and their associated uncertainty matrix:

$$p_{\min} = \begin{pmatrix} -4.001 \\ 30.999 \\ 10.003 \end{pmatrix} \mathrm{mm}.\ p_{\max} = \begin{pmatrix} 29.997 \\ 67.003 \\ 9.997 \end{pmatrix} \mathrm{mm}.$$

$$[p_i] = 10^{-6} \begin{pmatrix} 1.323 & 1.21 & 0.81 \\ 1.21 & 1.562 & 1.265 \\ 0.81 & 1.265 & 0.81 \end{pmatrix} \mathrm{mm}.$$

3.1 GUM application

The final uncertainty is obtained by propagating the elementary components through a linear approximation. Few simplifications can be done to calculate the Jacobian matrix:

$$\begin{array}{c} \dfrac{\partial df}{\partial n_x} = x_{p_{\min}} - x_{p_{\max}}; \quad \dfrac{\partial df}{\partial n_y} = y_{p_{\min}} - y_{p_{\max}}; \\ \dfrac{\partial df}{\partial n_z} = z_{p_{\min}} - z_{p_{\max}} \\ \dfrac{\partial df}{\partial x_{p_{\min}}} = n_x; \quad \dfrac{\partial df}{\partial x_{p_{\min}}} = n_y; \quad \dfrac{\partial df}{\partial x_{p_{\min}}} = n_z \\ \dfrac{\partial df}{\partial x_{p_{\max}}} = -n_x; \quad \dfrac{\partial df}{\partial x_{p_{\max}}} = -n_y; \quad \dfrac{\partial df}{\partial x_{p_{\max}}} = -n_z \end{array}.$$

The final form of the Jacobian matrix is given as follows:

$$J^T = \begin{pmatrix} x_{p_{\min}} - x_{p_{\max}} \\ y_{p_{\min}} - y_{p_{\max}} \\ z_{p_{\min}} - z_{p_{\max}} \\ n_x \\ n_y \\ n_z \\ -n_x \\ -n_y \\ -n_z \end{pmatrix} = \begin{pmatrix} -33.998 \\ -36.004 \\ 0.006 \\ 0.000101811 \\ -0.00010666 \\ 0.99999999 \\ -0.000101811 \\ 0.00010666 \\ -0.99999999 \end{pmatrix} \mathrm{mm}.$$

The final form of the variance covariance matrix is given as follows:

$$M = \begin{pmatrix} \begin{matrix} 1{,}666E-09 & -1.425E-09 & -5.071E-14 \\ -1.425E-09 & 3.6E-09 & 8.542E-14 \\ -5.071E-14 & 8.542E-14 & 5.067E-18 \end{matrix} & (0) & (0) \\ (0) & \begin{matrix} 1.323E-06 & 1.21E-06 & 8.1E-07 \\ 1.21E-06 & 1.562E-06 & 1.265E-06 \\ 8.1E-07 & 1.265E-06 & 8.1E-07 \end{matrix} & (0) \\ (0) & (0) & \begin{matrix} 1.323E-06 & 1.21E-06 & 8.1E-07 \\ 1.21E-06 & 1.562E-06 & 1.265E-06 \\ 8.1E-07 & 1.265E-06 & 8.1E-07 \end{matrix} \end{pmatrix} \mathrm{mm}.$$

which leads us to the parallelism error and its associated uncertainty:

$$\mathrm{d}p = |\vec{p_{\max}p_{\min}}.\vec{n_0}| = 0.0064\,\mathrm{mm}$$

$$U(\mathrm{d}p) = 2u_c(\mathrm{d}p) = 0.0044\,\mathrm{mm}$$

3.2 Monte Carlo simulation

To begin the simulation, we generated the parallelism equation on the *PFSoft-Algebraic Calculator* software.

$$\mathrm{d}p = |(x_{p_{\min}} - x_{p_{\max}})n_{xo} + \left(y_{p_{\min}} - y_{p_{\max}}\right)n_{yo} + (z_{p_{\min}} - z_{p_{\max}})n_{zo}|.$$

We then loaded the equation on the *Monte Carlo editor* software, based on the hypothesis that each of the parameters $(x_{p_{\min}}, y_{p_{\min}}, z_{p_{\min}}, x_{p_{\max}}, y_{p_{\max}}, z_{p_{\max}}, n_{xo}, n_{yo}, n_{zo})$ follow a normal distribution law with standard deviation $\boldsymbol{\sigma}$=U/k from their respective matrices with a coverage factor $k = 2$.

After generating a sample of size 10^6 we have obtained the following distribution function:

We obtain the following values: $\begin{cases} y_{low} = 0.00177627\,\mathrm{mm} \\ y_{mean} = 0.00637961\,\mathrm{mm} \\ y_{high} = 0.01097144\,\mathrm{mm} \\ \sigma = 0.00229789\,\mathrm{mm} \end{cases}$

Which leads us to the expanded parallelism associated uncertainty $U = 2\sigma = 0.00459566\,\mathrm{mm}$.

The numerical tolerance is $\zeta = 0.5E-3mm$, and (d_{low}, d_{high}) calculation is the following as they represent the limits for a 95.45% confidence interval ($\overline{y} \pm 2\sigma$) of the generated distribution:

$$\begin{cases} d_{low} = |\mathrm{d}p - U(\mathrm{d}p) - y_{low_{MCS}}| = |0.00637885 \\ \quad - 0,00434707 - 0.00177627|\mathrm{mm}. \\ d_{high} = |\mathrm{d}p + U(\mathrm{d}p) - y_{high_{MCS}}| = |0.00637885 \\ \quad + 0,00434707 - 0.01097144|\mathrm{mm}. \end{cases}$$

$$\begin{cases} d_{low} = 2.555E-4\mathrm{mm} \le \zeta \\ d_{high} = 2.2451E-4\mathrm{mm} \le \zeta \end{cases}$$

After comparison, the numerical tolerance is significantly higher than d_{low} *and* d_{high}, meaning that the validation criterion $max(d_{low}, d_{high}) \le \zeta$ is verified, thus the validation of our uncertainty estimation.

3.3 Inter-laboratory comparison

In order to determine whether our model is valid, an inter-laboratory comparison must be conducted. ILCs aim to determine the efficacy and suitability of a certain analytical technique, in our case, can be used to validate our model's results.

Laboratory: Measurement Control Center (MCC).
CMM used: Zeis Duramax.
Software: Calypso.
Measuring volume: $X = 500$ mm, $Y = 500$ mm, $Z = 500$ mm.
Probe type: Vast-xxt-tl3.
Stylus diameter: 2 mm.
Temp: 20 ± 2 °C.
$p_{\boldsymbol{L}} = 5.3$ µm.
$U_{\boldsymbol{L}} = 3.3$ µm.

While many methods of evaluation exist, the commonly used comparison method is the normalized error ($E_{\boldsymbol{n}}$) formula, calculated using this equation [16,17]:

$$E_n = \frac{|\mathrm{d}p - p_L|}{\sqrt{U(\mathrm{d}p)^2 + {U_L}^2}}, \tag{8}$$

where $p_{\boldsymbol{L}}$ is the parallelism error measured by the participant laboratory and $U_{\boldsymbol{L}}$ its associated measurement uncertainty. if $|\ E_n\ | \le 1$ the result is satisfactory.

$$E_n = \frac{|0.0064 - 0.0053|}{\sqrt{0.0044^2 + 0.0033^2}} = 0.2 < 1.$$

After calculation, the normalized error is significantly inferior to one, we can then conclude that our model based on the GUM is valid, and that the CMM used in the PCMT laboratory is accurate.

4 Conclusion

The present paper shows a different approach for estimating the uncertainty of parallelism error of an industrial part using CMM. We began by setting the mathematical model of the parallelism of the surface according to the ISO 1101 specifications. We proceeded next to the inspecting the tolerance surfaces by probing a set of data points and estimating their measurement uncertainties according to the ISO 10360 instead of using the MPE given by the constructor. We then proceeded to the creation of the fitted least square datum plane and its associated parameters uncertainties using the algorithm we coded on Matlab, which allowed us to estimate and compare the associated uncertainties using the GUM method and Monte Carlo method, both showing remarkably close results (gap less than 10^{-4} mm). We finally proceeded to an inter-laboratory comparison; The normalized error showed satisfactory results, which validates our developed model.

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