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# Compliance of low-cost, single-frequency GNSS receivers to standards consistent with ISO for control surveying

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**Abstract.** The emergence of single-frequency, navigation-type Global Navigation Satellite System receivers capable to provide carrier phase data [the so-called high sensitivity (HS) carrier phase positioning] has been steadily growing over the recent years. The main purpose of this study is to metrologically evaluate two low-cost, HS receivers, namely the u-blox LEA-6T and NEO-7P, in control surveying specifications. The evaluation was carried out within a published framework of standards and associated guidelines that are consistent with standards from the International Standards Organisation. The survey results were obtained from sufficient independent testing and proof and achieved an accuracy classification of '1 cm' at 95% confidence level. This indicates that the particular type of receiver used with geodetic antennas can provide positioning results for general purpose control surveying applications that are comparable to using geodetic receivers and with a significantly lower cost.

**Keywords:** low-cost GNSS receivers (Global Navigation Satellite System) / control surveying networks / standards / accuracy specifications

## 1 Introduction

Satellite positioning (GNSS – Global Navigation Satellite System) is an important technology which has profoundly influenced the profession of surveying as well as a plethora of modern society's economic, scientific and social activities. This is because surveying is a multi-disciplinary industry that is technologically advanced in its use of geospatial information systems. Surveying operates in a wide range of sectors including geodesy, mapping, land development, mining engineering, property development, hydrography, agriculture. All the above applications rely on survey control marks that form the basis of a country's geodetic framework. This framework provides the underlying control of position and elevation on which all surveying reference points are based. Fundamental geodetic data is required at the millimetre to centimetre level depending on the application, as discussed in Section 2.2, and is determined from systems such as GNSS. Besides the above, GNSS is already an important aspect of mass produced mobile devices (e.g. smartphones).

The evolution of GNSSs, with more systems available and enhancement of receiver technology and algorithms, is ongoing with a primary focus in reducing weight, size, power consumption and complexity for the user. With the advances in receiver hardware and data processing technologies, low-cost GNSS receivers have been developed to improve real-time positioning activities at an accuracy of few centimetres by the output of instant high quality carrier phase measurements. These so-called high sensitivity (HS) GNSS receivers have different receiver architecture to standard geodetic receivers allowing them to acquire signals from more satellites and reduce the time-to-first-fix (TTFF). The main advantages include their low-cost (in the order of few hundred euros) and ease to use as they are often assembled in 'evaluation kits'.

The exploitation of the low-cost receivers has attracted the attention of researchers and manufacturers around the world [e.g. 1–10] who have performed numerous tests in a variety of static and kinematic environments. However, there is a growing demand for the surveying community to ensure that the emerging technology of low-cost HS receivers can meet standards that guarantee reliable geodetic control work.

Official standards from the International Standards Organisation (ISO) for GNSS receivers exist only for real-time kinematic (RTK) positioning. Specifically, the ISO 17123 series "Optics and optical instruments – Field procedures for testing geodetic and surveying instruments – Part 8: GNSS field measurement systems in real-time kinematic (RTK)" first published in 2008 refer to specific guidelines for geodetic type receivers. ISO standards for GNSS static surveying and control surveys do not

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exist but many national bodies worldwide have published their own (e.g. ANZLIC in Australia, FGCC in USA, CSRC in State of California, Land Information in New Zealand, Natural Resources in Canada, Ordnance Survey in UK, etc.). The standards are seen as an expression of current geodetic control surveying capabilities and usually define specific accuracy classifications. They also include testing methods, proofs and documentation necessary to achieve acceptable geodetic control work. The standards are almost always attended by 'best practices' which are field procedures widely recognised as capable of achieving stated levels of accuracy. Best practices provide the surveyors with guidelines known to produce high quality work, but are not a mandate for methods and processes.

Whilst many researchers have performed numerous static positioning tests with low-cost receivers and assessed their results with nominal values [e.g. 11–14], there is no published work to the authors' knowledge on relating control surveying with this type of receiver to standards and specifications. This work can be considered as a continuation on previous work by the authors [15] in evaluating low-cost receivers based on official standards. In [15] the evaluation was performed using official ISO standards that exist only for RTK positioning and the results indicated that the precision of the tested receivers was satisfied for real-time positioning RT-Class 2 at 95% confidence level. Typical applications for RT-Class 2 include network densification control, topographic control, and utility stake out.

In light of the developments with low-cost, HS receivers, this work investigates the metrological assessment of this type of receiver for routine control surveying using static GNSS positioning by following published standards and specifications. Specifically, two low-cost receivers, namely u-blox NEO-7P and u-blox LEA-6T, attached with geodetic antennas are verified in a test control network. The experimental tests described in this work followed approved methodologies and adherence to best practise approach for testing GNSS equipment for surveying applications. The survey results were obtained from sufficient independent testing and proof following published guidelines and specifications that are consistent with ISO standards. The paper comprises 5 sections. In Section 2 a brief discussion about low-cost GNSS receivers is given along with a subsection on the standards and specifications used in this work. Section 3 describes the data collection and Section 4 provides an analysis of the results. The concluding remarks of this work are given in Section 5.

## 2 Background

### 2.1 Types of low-cost receivers

Low-cost GNSS positioning has spread in the last few years due to several technological advances, such as assisted positioning and large parallel correlation [16]. These technologies allowed for faster TTFF (i.e. faster positioning once the hardware is switched on) and lower hardware cost, causing GNSS chipsets to be embedded in almost all mobile devices.

Low-cost positioning started in the late 1990s using Original Equipment Manufacturer (OEM) type low-cost GPS receivers/boards that can output carrier phase data as an alternative to the geodetic receivers [e.g. 17–19]. These boards need to be integrated with logging devices such as a computer card and be driven by suitable software, thus hindering their wide use in routine surveying applications. Advancement to the boards is the small GNSS handheld modules with most popular being the Garmin-family receivers [e.g. 20–22]. These handheld receivers can track carrier phase data but do not output the raw data because there is no official interface provided for this. Once more, these receivers cannot be implemented in precise surveying applications.

A major progress in low-cost GNSS positioning came with the development of the so-called HS receivers which are capable of providing satellite measurements for signals attenuated by approximately 35 dB. In this category, there are receivers which only document their phase data internally, with no access of the raw data to users, such as the SiRF-chip receivers [e.g. 23]. On the other hand, a limited number of manufacturers, such as u-blox, give access to their receiver technical documentation and the raw phase data for use in various positioning applications [e.g. 24–26].

### 2.2 Standards and specifications

To ensure the success of a geodetic control network, the criteria that must be satisfied involve adherence to best practices for data collection, quantifying the quality of the results using sufficient independent testing based on standards, and preparing and archiving of documentation showing compliance with the implemented standards, specifications and best practices.

The methodology followed in this work is in compliance with the standards and specifications of the relevant official Australian [27] and Californian [28] documentation. These have been chosen due to their similar approach and thoroughness amongst many other internationally covering all the above criteria. Briefly, some general considerations are given here.

The types of accuracy classification are defined as the 'network accuracy' which measures the relationship between the control point in question and the realisation of the datum, and the 'local accuracy' which measures the positional accuracy relative to other points within the same network. Both accuracy standards are computed by random error propagation from a least squares adjustment at the 95% confidence level.

Local accuracy is intended to quantify the repeatability that a surveyor should expect when measuring between two adjacent points. In practice, the assessment typically results in small difference from the network accuracy value. For this reason, network accuracy is adopted herein as the most intuitive and useful metric for classification of geodetic control accuracy.

Both the Australian and Californian standards and specifications address procedures for achieving classifications from 0.5 cm to 10 cm (at 95% level), the upper and lower margins of which reach the practical limitations for

**Table 1.** Classification of a GNSS control network based on SU as described in the Australian standards [27].

| Classification | Survey uncertainty (SU) | |
|---|---|---|
| | Horizontal position (mm) | Ellipsoidal height (mm) |
| Below 15 mm (horiz.) and 20 mm (vert.) | <15 | <20 |
| Below 30 mm (horiz.) and 50 mm (vert.) | <30 | <50 |

**Table 2.** Classification of a GNSS control network based on the Californian standards [28].

| Classification | 95% confidence region (m) | Notes |
|---|---|---|
| 1 mm | ≤0.001 | Outside the scope of the specifications |
| 2 mm | ≤0.002 | |
| 0.5 cm | ≤0.005 | Included in the specifications |
| 1 cm | ≤0.01 | |
| 2 cm | ≤0.02 | |
| 5 cm | ≤0.05 | |
| 10 cm | ≤0.1 | |
| 2 dm | ≤0.2 | Outside the scope of the specifications |
| 5 dm | ≤0.5 | |
| 1 m | ≤1 | |

routine control surveying applications. Specifically, Table 1 gives the Australian classification in terms of survey uncertainty (SU) and Table 2 gives the Californian classification.

In Table 1, SU is defined as the uncertainty of the horizontal and/or vertical coordinates of a survey control mark independent of datum, thus is free from the influence of any imprecision or inaccuracy in the underlying datum realisation [27]. Therefore, SU reflects only the uncertainty resulting from survey measurements, measurement precisions, network geometry and the choice of constraint. A minimally constrained least squares adjustment is the preferred and most rigorous way to estimate and test SU at a specified confidence level (usually at 95%).

Based on the guidelines of the Surveyors Practice Directions of Australia [29], the surveyors are required to attend the following tests in order to test their GNSS equipment:
- A zero baseline test, which is used to determine the precision of the receiver measurements.
- A baseline comparison test, i.e. comparison of repeated GNSS derived distances against certified distances.
- A network test which includes at least three survey pillars (lengths vary from 150 m to 10 km).

The best practises in data collection always refer to using sound survey techniques to achieve the desired accuracies in a network. Also, it is important to perform GNSS observations at different sidereal times, with different satellite configurations, and different atmospheric conditions as the strongest defence against systematic errors and excessive random errors [30–33]. In addition, the recommendations include observation session length of 1 h/1 km and elevation mask above 5°.

## 3 Data collection

This section presents results from the implementation of the two low-cost receivers in a test control network. The equipment is described and the system configuration is explained in detail.

### 3.1 Equipment

The equipment employed in this work involves the low-cost single frequency receivers u-blox NEO-7P XXL and u-blox LEA-6T (www.ublox.com) which were used in conjunction with the geodetic antennas Javad JPS LegANT W/flat ground-plane (www.javad.com). The NEO-7P XXL receiver can obtain measurements from GPS and GLONASS satellites while the LEA-6T receiver can only receive GPS measurements. The receivers were connected to laptop computers for their operation and data archiving (Fig. 1).

Although it is best practice to use the antenna which is designed for the receiver model, in the control surveying tests described herein it was not possible to use navigation antennas mainly for phase centre issues. Also, the best practice guidelines do not recommend mixing different types of receivers even from the same manufacturer. In the tests of this work, mixing two types of low-cost receivers was inevitable due to limited number of available receivers. However, it was ensured that there was compatibility for both receivers regarding the firmware and the same type of geodetic antenna. According to the code of practise for GNSS observations, the antenna setup over the pillars was realised with forced centring devices [34] of high precision (order of 0.5 mm).

**Fig. 1.** Receiver configuration using the low-cost receivers (u-blox LEA-6T and NEO-7P) connected to geodetic antennas.

The results of the low-cost receivers were compared against the results from measurements of the same control network obtained with dual-frequency geodetic receivers. These were a set of two receivers Trimble 5800 with their antenna (quoted accuracy of 5 mm ± 0.5 ppm horizontally and 5 mm ± 1 ppm vertically).

The data acquisition was performed using the u-center program environment (www.u-blox.com). This is freeware software that provides a tool for configuration and performance analysis of the u-blox GNSS receivers.

## 3.2 Test description

The field tests comprised a zero baseline test, a pillar baseline comparison under different geometry, a 4-pillar control network and a baseline comparison of distances up to 18 km.

The 4-pillar network is a subset of a larger control network established in the university campus. All pillars are located on the roof tops of various university buildings. Although this is not an officially certified calibration network, it is extensively measured throughout a number of years with many different geodetic techniques providing positioning results of statistically high confidence (i.e. order of 1–2 mm at 95% confidence level). Figure 2a depicts (in Google Map) the four chosen pillars that were used as a test control network in this work, and Table 3 provides the approximate distances of the network baselines. For all the tests described below, best practises in data collection have been followed.

The zero-baseline test involved the two low-cost receivers (LEA-6T and NEO-7P) attached to the geodetic antenna through a special RF antenna splitter cable. The pillar LAMB (cf. Fig. 2a) was used for this test and the duration of the measurements was 24 h.

The baseline comparison involved pillars LAMB and ESTIA. The observation scheme included observations for approximately 1 h and, after a gap of about 90 min to ensure adequate change in the satellite constellation, a receiver swap was performed for another observation session of 1 h.

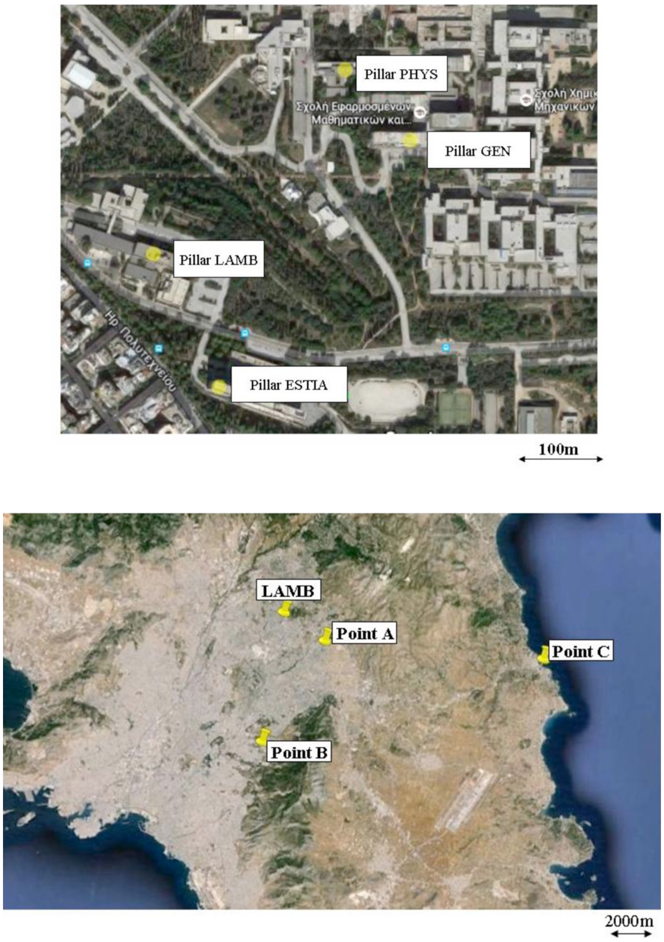

**Fig. 2.** (a) View of the test control network (Google map). (b) View of the long baseline test area (Google map).

**Table 3.** Baseline approximate distances.

| Baseline | Approximate distance (m) |
|---|---|
| LAMB-ESTIA | 165 |
| LAMB-PHYS | 300 |
| LAMB-GEN | 320 |
| GEN-PHYS | 110 |
| GEN-ESTIA | 350 |
| PHYS-ESTIA | 380 |
| LAMB-A | 3500 |
| LAMB-B | 9200 |
| LAMB-C | 18 700 |

The control network test involved observations to 12 independent baselines. This is in compliance with the best practice recommendations of the Australian and Californian standards regarding data collection (i.e. repeat station observations at two times for 100% of stations and at three times for 10% of the stations). The large number of independent baselines removes the possibility of highly correlated observations being

**Table 4.** Statistics for the zero baseline test.

| Receiver | Mean difference (m) | | | Standard deviation (m) | | |
|---|---|---|---|---|---|---|
| | ΔNorth | ΔEast | ΔUp | ΔNorth | ΔEast | ΔUp |
| u-blox LEA-6T | 0.001 | 0.000 | 0.000 | 0.001 | 0.001 | 0.001 |
| u-blox NEO-7P | 0.000 | 0.000 | 0.001 | 0.001 | 0.000 | 0.001 |

present in the network adjustment. The observation session length was in the order of 40–45 min at a rate of 1 s and with elevation angle of 5°. The number of visible satellites was always over 5.

The baseline lengths of the test network are perhaps shorter than those used in many surveying applications. For this reason, a further test was performed that examined the positioning accuracy in distances at the order of 5 km, 10 km and 18 km (Fig. 2b). In these tests, there were no fixed pillars used due to unavailability of pre-established pillars in the area. As base station, the pillar LAMB was used with one low-cost receiver collecting data. The rover station was positioned in distances up to 18 km and, for each station set-up, two receivers (one geodetic and the second low-cost) were connected to the geodetic antenna through a special RF antenna splitter cable. The observation session was in the order of 40–45 min at a rate of 1 s.

## 4 Data processing and analysis

The data processing for all collected data was performed using the GNSS post-processing software Grafnav and Grafnet (www.novatel.com). All the results were produced using double-differenced carrier phase observations and broadcast ephemeredes. The processing was intentionally performed with broadcast rather than precise ephemeredes because these are typically used in practical control surveying projects. The results were referred to the global geodetic reference system International Terrestrial Reference Frame 2008 (ITRF08) and hence to World Geodetic System 1984 (WGS84). The above results were also transformed to planimetric positions ($E$, $N$ – Easting, Northing) that refer to the plane projection (UTM – Universal Transverse Mercator) of the Greek Reference Datum 1987 (EGSA87). The orthometric heights referred to Earth Gravitational Model 2008 (EGM2008) geoid model.

The steps discussed below refer to (a) zero baseline test, (b) repeatable baseline processing, (c) network least squares solution (minimally constrained and fully constrained), (d) long baselines.

### 4.1 Zero baseline

The ‘zero baseline’ test was performed on January 2016 with the low-cost receiver NEO-7P logging data for 25 h 25 min and the receiver LEA-6T logging for 21 h 54 min. With this test all common errors due to multipath, noise, propagation effects, satellite orbits and clocks cancel in the processing. Both receivers obtained zero baseline phase results of better than 1 mm at 95% confidence level. It is seen from Table 4 that the two receivers have a very similar performance which indicates low receiver noise. Often, different types of receivers, even from the same manufacturer, may use different tracking algorithms and apply different data averaging times which can affect noise levels. The values in Table 4 show the differences ΔNorth, ΔEast, ΔUp in the baseline components between the geodetic and low-cost receivers with their associated standard deviations. It is seen that these are well within the specifications criteria, i.e. better than 3 mm at 95% confidence level [29].

### 4.2 Repeated baseline processing

A chosen baseline (“LAMB-ESTIA”) was observed four times for a duration of 40 min per observation session. For every one of the observation sessions a receiver swap was performed as per the guideline in [29] allowing a period of about 90 min for change of satellite configuration. Table 5a shows the results of the vector components with their associated standard deviations for the repeated observations using the low-cost receivers. Table 5b shows the respective results using the geodetic receivers. The baseline components $\Delta X$, $\Delta Y$, $\Delta Z$ refer to WGS84.

The baseline components from each type of receiver were compared not only against them but also against the ‘truth’. As discussed in Section 3.2, the pillar network has been measured extensively by a number of different geodetic techniques and thus, the pillar coordinates are considered as ‘truth’. In Tables 5a and 5b, the mean difference from the ‘truth’ is given. It is seen that the repeatability for both types of receivers is comparable. Specifically, as indicated by the standard deviation of the mean, the precision is 3 mm for the low-cost receivers and 2 mm for the geodetic receivers.

In order to verify this statistically, a two-tailed $f$-test was used to evaluate that the population standard deviations are equal (null hypothesis $H_0$) or not equal (alternative hypothesis $H_1$). Specifically, the values for the horizontal and vertical components of the low-cost receivers (Tab. 5a) are compared to the respective values for the horizontal and vertical components of the geodetic receivers (Tab. 5b). For a critical region of $a = 0.025$ (i.e. 95% level) the null hypothesis is rejected when $f < 0.10$ or $f > 9.97$. Because $f_{\text{horizontal}} = 1.38$ and $f_{\text{vertical}} = 1.5$ it is concluded that the null hypothesis cannot be rejected.

Figure 3 depicts the baseline repeatability with values up to 3 mm in the horizontal and up to 6 mm in the vertical for the low-cost receivers. For the geodetic receivers, the repeatability is 0.001 m for all three components.

**Table 5a.** Baseline repeatability (low-cost receivers).

| | | Baseline components (m) | | | | | |
|---|---|---|---|---|---|---|---|
| Baseline | | $\Delta X$ | $\Delta Y$ | $\Delta Z$ | $\sigma\Delta X$ | $\sigma\Delta Y$ | $\sigma\Delta Z$ |
| LAMB | ESTIA | 73.869 | 115.745 | −99.053 | 0.0006 | 0.0004 | 0.0017 |
| ESTIA | LAMB | 73.866 | 115.745 | −99.047 | 0.0009 | 0.0008 | 0.0011 |
| LAMB | ESTIA | 73.867 | 115.746 | −99.046 | 0.0011 | 0.0007 | 0.0011 |
| ESTIA | LAMB | 73.872 | 115.745 | −99.048 | 0.0008 | 0.0008 | 0.0012 |
| Mean from 'truth' | | 0.0007 | 0.0003 | 0.006 | | | |
| Stand. deviation | | 0.003 | 0.0007 | 0.003 | | | |

**Table 5b.** Baseline repeatability (geodetic receivers).

| | | Baseline components (m) | | | | | |
|---|---|---|---|---|---|---|---|
| Baseline | | $\Delta X$ | $\Delta Y$ | $\Delta Z$ | $\sigma\Delta X$ | $\sigma\Delta Y$ | $\sigma\Delta Z$ |
| LAMB | ESTIA | 73.870 | 115.745 | −99.047 | 0.0005 | 0.0004 | 0.0009 |
| ESTIA | LAMB | 73.869 | 115.747 | −99.050 | 0.0007 | 0.0006 | 0.0010 |
| LAMB | ESTIA | 73.870 | 115.746 | −99.046 | 0.0007 | 0.0007 | 0.0010 |
| ESTIA | LAMB | 73.871 | 115.747 | −99.048 | 0.0008 | 0.0007 | 0.0009 |
| Mean from 'truth' | | 0.000 | 0.00017 | 0.0001 | | | |
| Stand. deviation | | 0.001 | 0.0006 | 0.002 | | | |

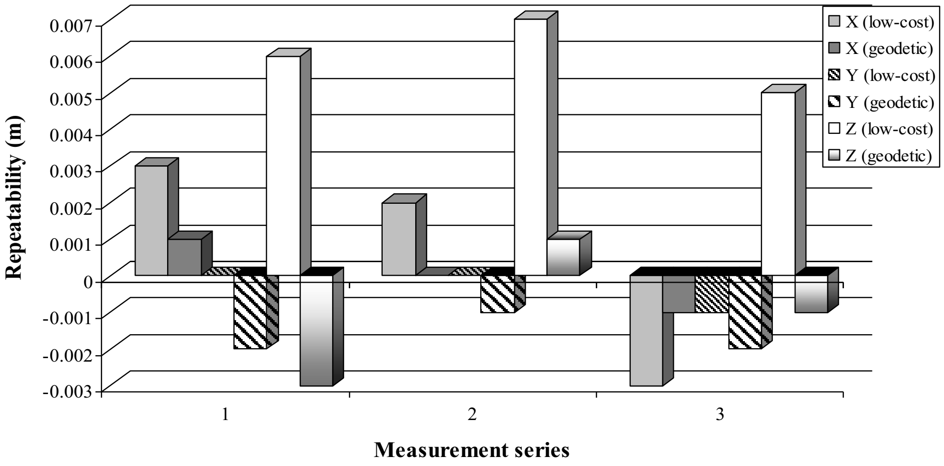

**Fig. 3.** Baseline repeatability.

## 4.3 Network processing

This part of the analysis refers to the network data which are processed initially on a baseline-by-baseline basis and then a least squares adjustment is performed to assess the quality of the survey data as a whole.

The baseline processing was performed separately and, prior to any further processing, loop closure checks were made to eliminate blunders. Table 6 provides the closure errors in $X$, $Y$, $Z$ and the total closure error ($C_{\mathrm{Error}}$). The latter is computed as the square root of the sum of the squared $C_X$, $C_Y$, $C_Z$. It is seen that $C_{\mathrm{Error}}$ is not exceeding 3–4 mm for the low-cost receivers which indicates that no gross errors exist in the processed baselines.

Table 7 provides the absolute differences in each baseline component obtained from the two types of receivers (low-cost vs. geodetic). It is seen that the mean differences vary up to 5 mm.

Following the independent baseline processing, a network processing was performed using a minimally constrained adjustment (LAMB station fixed). The solution involved 42 measurements [degrees of freedom $(\mathrm{DoF}) = 33$] and the a posteriori variance factor was estimated as 1.002 (the a priori sigma was set to unity). Table 8a gives, for both types of receivers, the final adjusted planar coordinates in WGS84 and Table 8b gives the same coordinates in the plane projection of EGSA87. The adjusted results are shown in both the geocentric

**Table 6.** Loop closure errors.

| Loop | LAMB-PHYS-GEN-LAMB | | PHYS-GEN-ESTIA-LAMB | | LAMB-ESTIA-GEN-LAMB | |
|---|---|---|---|---|---|---|
| | Low-cost | Geodetic | Low-cost | Geodetic | Low-cost | Geodetic |
| $C_X$ (m) | −0.004 | 0.002 | −0.002 | −0.002 | 0.002 | 0.002 |
| $C_Y$ (m) | 0.001 | 0.000 | −0.002 | 0.000 | 0.002 | 0.000 |
| $C_Z$ (m) | −0.002 | 0.000 | −0.002 | −0.003 | −0.001 | −0.004 |
| $C_{\text{Error}}$ (m) | 0.004 | 0.002 | 0.003 | 0.003 | 0.004 | 0.004 |

**Table 7.** Differences in baseline components (low-cost vs. geodetic receivers).

| Baseline | | $\Delta X^{\text{geod}}-\Delta X^{\text{low-cost}}$ (m) | $\Delta Y^{\text{geod}}-\Delta Y^{\text{low-cost}}$ (m) | $\Delta Z^{\text{geod}}-\Delta Z^{\text{low-cost}}$ (m) |
|---|---|---|---|---|
| ESTIA | PHYS | 0.001 | 0.0001 | 0.004 |
| PHYS | GEN | 0.007 | 0.000 | 0.005 |
| GEN | ESTIA | 0.005 | 0.001 | 0.009 |
| GEN | LAMB | 0.001 | 0.001 | 0.006 |
| LAMB | ESTIA | 0.001 | 0.002 | 0.002 |
| LAMB | PHYS | 0.000 | 0.000 | 0.001 |
| Mean ± stand. deviation | | 0.002 ± 0.003 | 0.001 ± 0.001 | 0.005 ± 0.003 |

**Table 8a.** Adjusted network coordinates in WGS84 (minimally constrained solution).

| Pillar | Receiver | Adjusted coordinates | | | Abs. differences | | |
|---|---|---|---|---|---|---|---|
| | | $X$ (m) | $Y$ (m) | $Z$ (m) | $\Delta X$ (m) | $\Delta Y$ (m) | $\Delta Z$ (m) |
| ESTIA | Low-cost | 4,606,976,169 ± 0.003 | 2,030,090.740 ± 0.002 | 3,903,328.293 ± 0.002 | 0.003 | 0.002 | 0.006 |
| | Geodetic | 4,606,976.166 ± 0.001 | 2,030,090.738 ± 0.001 | 3,903,328.287 ± 0.003 | | | |
| PHYS | Low-cost | 4,606,694.955 ± 0.003 | 2,030,123.780 ± 0.002 | 3,903,585.544 ± 0.002 | 0.003 | 0.000 | 0.002 |
| | Geodetic | 4,606,694.958 ± 0.001 | 2,030,123.780 ± 0.001 | 3,903,585.542 ± 0.002 | | | |
| GEN | Low-cost | 4,606,719.073 ± 0.002 | 2,030,217.187 ± 0.002 | 3,903,529.898 ± 0.002 | 0.006 | 0.004 | 0.005 |
| | Geodetic | 4,606,719.067 ± 0.001 | 2,030,217.183 ± 0.001 | 3,903,529.893 ± 0.003 | | | |

reference datum and the national grid to verify that the national datum does not impose any evident errors in the solutions. The differences do not exceed few mm in the horizontal and just above 1 cm in the vertical components. The accuracy of the measured network is 0.0013 m in the horizontal and 0.01 m in the vertical, which provides an overall accuracy of 0.01 m. Thus, the network is classified as category '1 cm' at 95% confidence level (cf. Tab. 2). Also, it fits in the first category of Table 1 (i.e. $\text{SU}_{\text{horizontal}} < 15$ mm, $\text{SU}_{\text{vertical}} < 20$ mm).

In order to evaluate the SU, the Australian guidelines define that a 'local test' procedure should be performed to assess the quality of a measurement and its assumed uncertainty [28]. To validate each measurement (in this case the components of each baseline) and its uncertainty, the size of each adjusted measurement correction is tested to verify that the correction lies within the upper and lower limits of the specified confidence interval, i.e. at 95%. Specifically, the normalised residuals for each baseline are checked whether they exceed the critical value of the unit normal distribution at 95%. Table 9 gives the relevant information for the three components $X$, $Y$, $Z$ (in WGS84) for the baselines measured by the low-cost receivers.

From Table 9, it is seen that a small number of normalised residuals exceed marginally the critical value of the unit normal distribution at 95% (i.e. 1.96). All observations were taken under the same set of conditions and therefore these marginal failures are most likely the consequence of over-optimistic measurement precisions, rather than the specific baselines containing gross errors. However, re-scaling of all measurements had no effect on the a posteriori variance factor of the minimally

**Table 8b.** Adjusted network coordinates in planar projection of EGSA87 (minimally constrained solution).

| Pillar | Receiver | Adjusted coordinates | | | Abs. differences | | |
|---|---|---|---|---|---|---|---|
| | | *N* (m) | *E* (m) | Up (m) | Δ*N* (m) | Δ*E* (m) | ΔUp (m) |
| ESTIA | Low-cost | 4,202,651.710 ± 0.002 | 480,613.729 ± 0.001 | 275.646 ± 0.004 | 0.001 | 0.001 | 0.004 |
| | Geodetic | 4,202,651.709 ± 0.001 | 480,613.730 ± 0.001 | 275.650 ± 0.003 | | | |
| PHYS | Low-cost | 4,203,004.147 ± 0.002 | 480,758.127 ± 0.001 | 241.587 ± 0.003 | 0.005 | 0.003 | 0.003 |
| | Geodetic | 4,203,004.152 ± 0.001 | 480,758.130 ± 0.001 | 241.590 ± 0.002 | | | |
| GEN | Low-cost | 4,202,923.383 ± 0.002 | 480,833.655 ± 0.002 | 254.428 ± 0.004 | 0.006 | 0.000 | 0.007 |
| | Geodetic | 4,202,923.389 ± 0.001 | 480,833.655 ± 0.001 | 254.435 ± 0.003 | | | |

**Table 9.** Local test performed in baseline components (minimally constrained solution).

| Baseline | *X* correction (m) | *Y* correction (m) | *Z* correction (m) | *X* normalised residual | *Y* normalised residual | *Z* normalised residual |
|---|---|---|---|---|---|---|
| LAMB-ESTIA | 0.002 | 0.001 | 0.000 | 1.00 | 0.50 | 0.00 |
| LAMB-PHYS | 0.000 | −0.002 | 0.001 | 0.00 | −2.00 | 1.00 |
| LAMB-GEN | 0.002 | 0.000 | 0.000 | 1.00 | 0.00 | 0.00 |
| ESTIA-PHYS | 0.002 | 0.001 | −0.001 | −2.00 | 0.50 | 0.50 |
| GEN-ESTIA | 0.002 | 0.000 | −0.001 | −1.00 | 0.00 | −1.00 |
| GEN-PHYS | −0.002 | −0.001 | 0.001 | 2.00 | −0.50 | −0.50 |

constrained adjustment, thus indicating that the system of measurements is precise. Therefore, a fully constrained adjustment followed as discussed below.

The fully constrained least squares solution was performed to propagate datum and uncertainty with two stations held fixed (LAMB and ESTIA). The solution involved 42 measurements (DoF = 36) and the a posteriori variance factor was estimated as 1.002 (the a priori sigma was set to unity). The constrained adjustment at the 95% confidence level yields results that pass the local and global test as described in [28]. Table 10 provides the differences, which do not exceed 5 mm, between the minimally and fully constrained adjusted solutions for the two adjusted stations. Thus, it seems that the measurements as taken by the low-cost receivers were correctly indicated by the prescribed uncertainties, and none of the constraints was shown to bias the adjustment in a significant way or to cause any of the measurements to fail.

A measure of the network quality can be indicated by the 95% positional uncertainty (PU) and circular radius values (Tab. 11) that comprise the 'global' test defined by the Australian specifications. PU is defined as the uncertainty of the horizontal and/or vertical coordinates of a control point with respect to datum [28]. PU includes SU as well as the uncertainty of the existing survey control marks to which any new control survey is connected. A fully constrained least squares adjustment is the preferred and most rigorous way to estimate and test PU at a specified confidence level.

**Table 10.** Differences in the North, East, Up components between minimally constrained and fully constrained solutions.

| Station | ΔNorthing (m) | ΔEasting (m) | ΔUp (m) |
|---|---|---|---|
| PHYS | 0.004 | 0.001 | 0.001 |
| GEN | 0.002 | 0.002 | 0.005 |

**Table 11.** 95% PU and circular radius values (fully constrained solution).

| Station | PU (Northing) (m) | PU (Easting) (m) | PU (Up) (m) | Circular radius (m) |
|---|---|---|---|---|
| PHYS | 0.006 | 0.005 | 0.006 | 0.006 |
| GEN | 0.005 | 0.005 | 0.006 | 0.006 |

Based on the estimated PU values, it can be assessed whether the survey control network has achieved any predefined uncertainty or quality threshold. From the foregoing analysis, the adjustment has proven to be successful and the network is classified as category '1 cm' at 95% confidence level (cf. Tab. 2). It also fits in the first category of Table 1 (i.e. $SU_{horizontal} < 15$ mm, $SU_{vertical} < 20$ mm).

Finally, to express uncertainty in terms of horizontal circular confidence region, the 95% uncertainty value is calculated from the standard (1 sigma) error ellipse and is expressed as a single quantity, being the radius of the circular confidence region. Table 12 gives for each adjusted baseline the radius of the circular region for the fully constrained adjustment [28]. The values for the semi-major axis $a$ and semi-minor axis $b$ of the standard error ellipse are derived from the full a posteriori variance–covariance matrix after the least squares adjustment. The value $K$ is the coverage factor for expressing appropriately the one-, two- or three-dimensional components at the 95% confidence level. The values shown in the table are of the same magnitude indicating that the network portrays the same pattern of precision and of significance.

**Table 12.** Horizontal circular confidence region at 95% confidence level (fully constrained solution).

| Baseline | $r = a \times K$ |
|---|---|
| LAMB-PHYS | 10.02 |
| LAMB-GEN | 4.77 |
| ESTIA-PHYS | 7.24 |
| GEN-ESTIA | 8.92 |
| GEN-PHYS | 7.02 |

### 4.4 Long baselines

The last part of the analysis refers to the long baseline data. Low-cost receivers are not very accurate in resolving long baselines (>12–15 km) where ionospheric effects have a larger impact on calculations [e.g. 30]. Table 13 provides the results of the positioning tests from the low-cost and the geodetic receivers both attached to the geodetic antenna. It is seen that the performance of the low-cost receiver is fairly consistent for all three baselines with the maximum differences not exceeding 2 cm.

In order to evaluate the measurement uncertainty (standard deviations) of the two populations (i.e. geodetic receivers and low-cost receivers), a two-tailed $f$-test was used to assess whether the population standard deviations (or variances) are equal (null hypothesis $H_0$) or not equal (alternative hypothesis $H_1$). Specifically, the values for the horizontal and vertical components of the low-cost receivers are compared to the respective values for the horizontal and vertical components of the geodetic receivers (given in Tab. 13). Table 14 provides the results for a critical region of $a = 0.025$ (i.e. 95% confidence level).

It is concluded from Table 14, that for the horizontal component, the null hypothesis cannot be rejected because there is insufficient evidence that the standard deviations of the two populations differ. On the other hand, for the vertical component, the null hypothesis cannot be accepted.

**Table 13.** Positioning results for long baselines.

| Control point | Receiver | Coordinates | | | Abs. differences | | |
|---|---|---|---|---|---|---|---|
| | | North (m) | East (m) | Up (m) | $\Delta N$ (m) | $\Delta E$ (m) | $\Delta$Up (m) |
| A | Low-cost | 4,209,628.199 ± 0.020 | 485,400.426 ± 0.020 | 250.084 ± 0.02 | 0.011 | 0.010 | 0.016 |
| | Geodetic | 4,209,628.188 ± 0.002 | 485,400.416 ± 0.002 | 250.068 ± 0.002 | | | |
| B | Low-cost | 4,202,800.617 ± 0.030 | 480,537.220 ± 0.020 | 208.100 ± 0.040 | 0.013 | 0.020 | 0.018 |
| | Geodetic | 4,202,800.650 ± 0.002 | 480,537.002 ± 0.002 | 208.118 ± 0.003 | | | |
| C | Low-cost | 4,207,504.335 ± 0.040 | 500,821.898 ± 0.020 | 4.199 ± 0.040 | 0.019 | 0.014 | 0.015 |
| | Geodetic | 4,207,504.316 ± 0.003 | 500,821.912 ± 0.002 | 4.214 ± 0.003 | | | |

**Table 14.** Statistical evaluation of measurement uncertainty for the long baseline test.

| Null hypothesis | Alternative hypothesis | $f_{\text{value}}$ ($\sigma_{\text{geod}}/\sigma_{\text{low-cost}}$) | | |
|---|---|---|---|---|
| | | A | B | C |
| $\sigma_{\text{geod}}^{\text{horiz}} = \sigma_{\text{low-cost}}^{\text{horiz}}$ | $\sigma_{\text{geod}}^{\text{horiz}} \neq \sigma_{\text{low-cost}}^{\text{horiz}}$ | $f_{\text{horiz}} = 0.01$<br>$0.025 < f_{\text{horiz}} < 39$ | $f_{\text{horiz}} = 0.006$<br>$0.025 < f_{\text{horiz}} < 39$ | $f_{\text{horiz}} = 0.0065$<br>$0.025 < f_{\text{horiz}} < 39$ |
| $\sigma_{\text{geod}}^{\text{vert}} = \sigma_{\text{low-cost}}^{\text{vert}}$ | $\sigma_{\text{geod}}^{\text{vert}} \neq \sigma_{\text{low-cost}}^{\text{vert}}$ | $f_{\text{vert}} = 0.1$<br>$f_{\text{vert}} < 39$ | $f_{\text{vert}} = 0.075$<br>$f_{\text{vert}} < 39$ | $f_{\text{vert}} = 0.075$<br>$f_{\text{vert}} < 39$ |

Clearly, more tests are needed using the low-cost receivers in long baselines in order to obtain statistically significant conclusions.

### 4.5 Discussion

In this study, low-cost single frequency receivers were evaluated in terms of accuracy specifications for control surveying. Single frequency receivers is not a new technology, they have been used since the inception of GPS. When used in static positioning and the phase ambiguities on L1 are correctly resolved, the precision obtained from a GNSS-L1 receiver is equivalent to that of an L1/L2 receiver for baseline distances up to 10–15 km. In fact, even for measurements from dual frequency receivers, most commercial software uses the L1 measurements to reduce the noise in the signal and the L2 measurements to correct cycle slips and propagation errors. Therefore, when using single frequency receivers for control survey network applications, the only actions available to the user are to use geodetic antennas and to ensure measurements under good observation conditions (i.e. observation sessions of longer than 15 min, maintaining low PDOP – position dilution of precision and track at least five satellites).

The evaluation of the low-cost receivers involved a zero-baseline test and network baseline measurements. The zero-baseline test represents the best possible solution that can be achieved with two receivers, as the configuration forces all allegedly common errors to be equal (i.e. satellite and atmospheric errors) and therefore, cancelling out completely. In an ideal case one could assume that noise and multipath could also cancel out, as the same signals are applied to both receivers. The processing showed that the two different low-cost receivers produced results in the order of 1 mm which is below the specifications of the 3 mm at 95% confidence level.

In terms of repeatability, the low-cost receivers produced baseline results within the accuracy specifications of geodetic receivers. The relative uncertainty (RU) for the same baseline did not exceed 1 mm in the horizontal and 6 mm in the vertical components. When this is related to a proportional form, the low-cost receivers provide an accuracy of $1.8 \times 10^{-5}$ ppm in the horizontal and $8 \times 10^{-5}$ ppm in the vertical versus the geodetic receivers which provide an accuracy of $4.5 \times 10^{-6}$ ppm in the horizontal and $1.2 \times 10^{-5}$ ppm in the vertical. It is seen that the low-cost receivers are well within the manufacturers' specifications for geodetic receivers. It is noted that the low-cost receivers have no accuracy specifications for static positioning by the manufacturer because they are navigation-grade receivers.

When reviewing the results of Tables 8a and 8b, it can be inferred that the results of the low-cost receivers are comparable to results from geodetic receivers. Also, there does not appear to be any degradation due to datum propagation and its uncertainty. The results in either WGS84 or the national grid are consistently precise. The low-cost receiver accuracy in $X$, $Y$, $Z$ is 0.004 m, 0.002 m, 0.004 m respectively with associated standard deviation of 0.002 m in all three components. Similarly, the low-cost receiver accuracy is 0.004 m, 0.001 m, 0.004 m in the three components with associated standard deviation of 0.002 m in all three components. It is seen that the horizontal coordinates resulting from the minimally constrained adjustment agree well within the acceptance criteria of 10 mm ± 15 ppm at the 95% confidence interval [28]. All the above indicates that the system of survey measurements, uncertainties and constraints is statistically reliable.

The results for the fully constrained adjustment indicate again a reliable network with positional uncertainties not exceeding 0.005 m in all components. This is somewhat expected because the baseline distances are small (no more than 0.5 km). The baselines measurements of 5 km, 10 km and 18 km provided results that when compared to the results of the geodetic receivers give a mean and associated standard deviation of 0.014 ± 0.004 m in $N$, 0.015 ± 0.005 m in $E$ and 0.0163 ± 0.002 m in the Up direction. From these values it can be inferred that for baseline lengths up to 18 km, the classification is at 2 cm (i.e. 0.02 m at 95% confidence level, cf. Tab. 2).

The main factors influencing the cost of a control survey project are capital cost of equipment, number of points to be surveyed, time taken to complete the necessary fieldwork, time to process the data and evaluate the results [31]. Considering the results shown in the previous sections and the fact that the hardware cost (order of few hundred euros) is significantly less than for standard geodetic receivers, these low-cost receivers are ideal for control surveys in the accuracy order of 0.01 m (at 95% confidence level). The requirement is use of a geodetic antenna and a distance limitation that is imposed by the single frequency observations.

## 5 Concluding remarks

GNSS innovation for surveying and mapping is seen in recent years mainly in terms of faster and more easily validated positioning with decreasing costs and, less about improved accuracy and precision. Whilst the accuracy needs within the surveying and mapping community for the most part have been met, there is still a need to evaluate new equipment and promote high quality practises.

This work demonstrated the evaluation of two low-cost receivers with geodetic antennas, in control surveying. It was not aimed to be a mere performance assessment of low-cost receivers in varying distances, but rather relating the performance uncertainty as a basis for evaluating and expressing the quality of the derived measurements and positions. The evaluation was carried out within a published framework of standards and associated guidelines whose definition of uncertainty is consistent with the ISO description.

The experimental tests employed in this work followed approved methodologies and adherence to best practise approach for testing GNSS equipment in surveying applications. This is essential when trying to be within acceptable statistical limits so that the results obtained for testing GNSS equipment comply with national and international standards.

The survey results were obtained from sufficient independent testing and proof and achieved an accuracy classification of '1 cm' at 95% confidence level (as per the Californian standards) and equivalent to the category of $\mathrm{SU}_{\mathrm{horizontal}} < 15\,\mathrm{mm}$ and $\mathrm{SU}_{\mathrm{vertical}} < 20\,\mathrm{mm}$ (as per the Australian standards).

So, are the low-cost receivers the future of GNSS positioning? This type of receiver used in conjunction with geodetic antennas and suitable software can meet the accuracy levels of 1–2 cm (at 95% confidence level) required for general purpose control surveying applications (e. g. network densification, land boundaries, stake out, digital mapping, etc.). For the high precision segment however (e.g. long-baseline static GNSS and safety critical applications) it is expected that the geodetic receiver market will continue to innovate and answer to a more demanding user community. With the benefits of having three operational GNSS systems, the innovations are seen mainly in the robustness for autonomous integrity monitoring which is of special importance in aviation and the ability to operate in obstructed environments including simplicity to receiver architecture.

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