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Cost comparisons of modified $\overline{X}$ chart for autocorrelated observations

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Abstract. This paper presents the economic design of modified $\overline{X}$ chart for autocorrelated data and comparison with the economic design of Shewhart's $\overline{X}$ chart. An attempt has been made to counter autocorrelation by designing the modified $\overline{X}$ chart; as the cost of operating a process control system is an important element in the economic design of control charts. The economic designs of both modified and Shewhart $\overline{X}$ charts for autocorrelated observations are presented; using Lorenzen-Vance [1] cost model. The modified $\overline{X}$ chart is based upon sum of chi-squares and has simplicity like Shewhart $\overline{X}$ chart but more efficient than standard chart. Two sets of parameters suggested by Montgomery [2] are used to determine the optimal sampling intervals (h) and expected costs per hour. It shows that sampling interval (h) and expected cost/hour (C) depend upon various parameters of chart, used in this model. The expected costs/hour (C) for both the charts are computed for sample sizes of 2 and 4 at the various levels of correlation and process shifts in mean. It is concluded that at each level of correlation and shift in the process mean, the expected costs/hour of the modified $\overline{X}$ chart are lower than the Shewhart chart.

Keywords: Level of autocorrelation, modified $\overline{X}$ chart, Lorenzen-Vance (1986) cost model, sample size, sampling intervals

1 Introduction

When a control chart is applied to monitor a process, some test parameters; such as the sample size, the sampling interval between successive samples, and the control limits should be determined. The real or assignable cause/s present in the process may lead to shift in the process mean or dispersion. For many processes; the observations are correlated and when this correlation build-up automatically in the entire process, is known as autocorrelation. Normally, it is assumed that the observations from the process output are independent and identically distributed (IID) but if these observations are serially correlated, the performance of control charts will be suspected because of more number of false alarms (type I error). Autocorrelation is inherent to many processes like in chemical, manufacturing, and service processes (where, manufacturing cycle time of products is small).

It is observed by the various researchers that the autocorrelation exists in the process output of the industries. Maragah and Woodall [3] provided results on the effect of autocorrelation on the performance of the Shewhart individuals control chart. Alwan and Roberts [4] found that more than 70% of the studied processes subject to change detection in quality control are autocorrelated. Moreover, in practice the positive type autocorrelation is more prevalent compared to the negative type. Figure 1 shows successive negatively correlated observations.

If the current observation is on one side of the mean, the next observation will most likely be found on the same side of the mean in case of positive type correlation. Positively correlated data are characterized by runs above and below the mean. According to Woodall and Faltin (1993), the positive correlation is more often encountered in practice than negative autocorrelation. They further investigated that positive autocorrelation becomes more of a problem when frequency of sampling increases, due to natural continuity common in industrial processes. It may also occur due to mixing of raw materials in a container. Figure 2 shows successive positively correlated observations.

The economic design of modified $\overline{X}$ chart for autocorrelated observations has been presented in this paper and the same has been compared with the economic design of Shewhart's $\overline{X}$ chart. As the cost of operating a process control system is an important element in the economic design of control charts; an attempt has been made to counter autocorrelation by designing the modified $\overline{X}$ chart on economic ground. The economic designs of both modified and Shewhart $\overline{X}$ charts for autocorrelated observations are presented; using Lorenzen-Vance [1] cost model.

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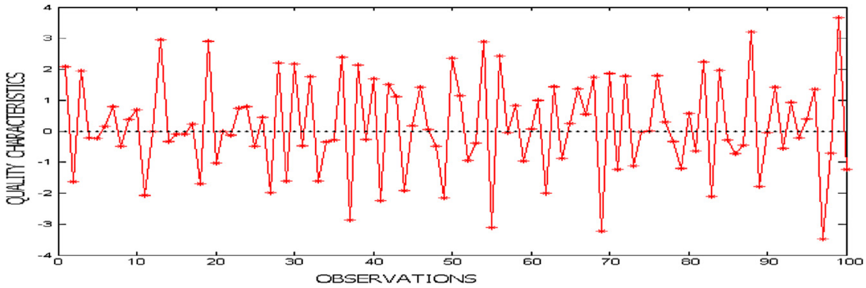

Fig. 1. Negative correlated observation.

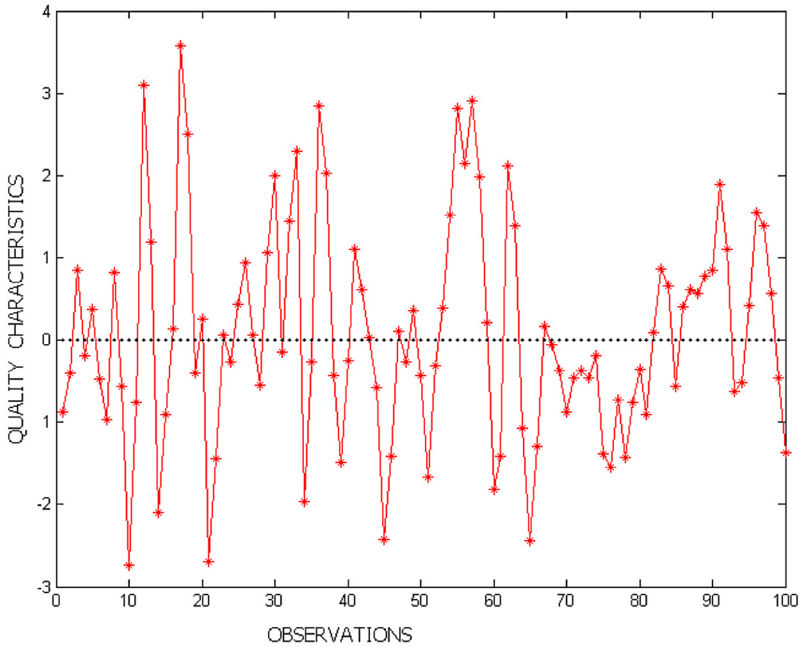

Fig. 2. Positive correlated observations.

The purpose of this paper is to show that the modified $\overline{X}$ chart is superior to Shewhart $\overline{X}$ on economic ground also. The comparison of performance on the basis of average run lengths (ARLs) of the modified $\overline{X}$ chart with the Shewhart $\overline{X}$ chart for correlated data has already been given in Singh and Prajapati [5].

The economic design of modified $\overline{X}$ chart, based on sum of Chi-squares theory is presented in this paper. Considerable attention has been given to the design of control charts on economic grounds by many researchers. Two different manufacturing process models are often cited in the literature of the economic design of control charts. Duncan's original work assumes that the process is not stopped when the investigation of a possible special cause is undertaken while some others models assume the process is stopped.

Most of the work of the economic design of quality control charts assumes that the underlying distribution of the process failure is exponential i.e. the times between occurrences of successive special causes are exponentially distributed with a specified mean value, and thus, a constant failure rate for the process is implied. For some processes that deteriorate with time, the exponential assumption may not be appropriate. Various researchers have worked for such situation and their work is discussed in this section. Duncan [6] presented the first cost model for economic design determining the values of test parameters for the $\overline{X}$ control charts that minimizes the average cost when a single out-of-control state (assignable cause) exists, which is called the economic design of $\overline{X}$ control charts. Since then, considerable attention has been devoted to the economic determination of the test parameters of $\overline{X}$ charts [7–10]. Lorenzen and Vance [1] provided a unified approach to the economic design of process control charts. They considered a general process model that applied to all control charts regardless of statistic used. McWilliams [11] developed an algorithm that enables users to determine economic, statistical, or economic-statistical $\overline{X}$ chart design. Economic designs are designs that minimize cost as measured by the Lorenzen-Vance [1] cost function. Statistical designs are designs that satisfy a series of average run length constraints associated with various degrees of shift in process mean. Hawkins and Zamba [12] concluded that the Shewhart $\overline{X}$ chart is effective in detecting large shifts in a process but unable to find moderate shifts.

Yu and Chen [13] provided an economic model to determine the optimal parameters of moving average (MA) control chart. Dias and Infante [14] investigated a new sampling methodology for systems with a known lifetime distribution, known as the Predetermined Sampling Intervals (PSI) method. Torng et al. [15] modified the Duncan's cost model by adding the statistical constraints to develop the design model of double sampling (DS) $\overline{X}$ chart for the optimization of design parameters-sample size, control limit coefficient, warning limit coefficient and sampling interval. Prajapati and Mahapatra [16] compared the economic design of modified $\overline{X}$ chart with the economic and economic-statistical design of multivariate exponentially weighted moving average (MEWMA) control chart modified by Linderman and Love [17]. Costa and Machado [18] compared the variable parameters (VP) $\overline{X}$ chart and double sampling (DS) $\overline{X}$ chart, at various shifts in process mean and two levels of correlation i.e. $\varphi = 0.4$ and 0.8. Fons [19] discussed the methods to quantify effects of having a quality management system, in monetary terms that can help to make decision making easier by company managers easier. They used various tools which can maximize their individual benefits from a holistic point of view of total quality. Yeong et al. [20] proposed an economic model for the synthetic chart that is an integration of the $\overline{X}$ chart and the *CRL* chart. Numerical examples, based on different values of input parameters were given, and sensitivity analyses of the parameters were performed. The input parameters which have a significant impact on the cost and choice of optimal parameters of the synthetic chart were identified by them. The effect of incorrect estimation of the input parameters was also investigated. They showed that if the chart cannot be operated at the economically optimal level, there is still a large choice of parameters to choose from which does not result in a large increase in cost. They compared their model with the synthetic Shewhart $\overline{X}$ and EWMA charts. Lin et al. [21] stated that the presence of autocorrelation in the process data can result in significant effect on the statistical performance of control charts. Schoonhoven and Does (2013) studies alternative standard deviation estimators that serve as a basis to determine the control chart limits used for real-time process monitoring (phase II).

Chopra and Garg [22] developed two simple models in the field of cost of quality. One for estimating/calculating cost of quality and the second model for implementing cost of quality system in an industry. They observed that by implementing same, the cost of quality reduces significantly in the chosen industry. Singh and Prajapati [5] suggested the optimal schemes of $\overline{X}$ chart and compared with VP $\overline{X}$ chart and DS $\overline{X}$ chart, suggested by Costa and Machado [18]. Mitra and Clark [23] focused on determining changes in process variability of multivariate processes. A couple of aggregate measures were modified and the performance of these suggested measures was explored through a simulation procedure. Yilmaz and Burnak [24] developed a mathematical model for the economic design of the cumulative count of conforming (CCC) control chart and presented an application of the proposed model. On the basis of the results of the application, the economic and classical CCC control chart designs of the CCC control chart are compared. The optimal design parameters for different defective fractions are tabulated, and a sensitivity analysis of the model was presented for the CCC control chart user to determine the optimal economic design parameters and minimum hourly costs for one production run according to different defective fractions, cost, time, and process parameters.

Lupo [25] proposed a multi-objective economic-statistical design approach for an adaptive $\overline{X}$ chart. His approach aims at the minimization of both the total quality related costs and the out-of-control average run length, in such a way assuring an optimal trade-off between economic and statistical performance of the related control procedure. He formulated a mixed integer nonlinear constrained mathematical model to solve the treated problem, whereas the Pareto optimal frontier was described by the ε-constraint method. Saghaei et al. [26] modeled the cost function of exponentially weighted moving average (EWMA) control chart by considering the measurement error and Taguchi loss functions for poor quality products. They computed the average run length by using Markov chain method, and finally, optimal values of parameters were obtained using genetic algorithm. They also applied sensitivity analysis of parameters and their results indicate that when the slope of covariate function increases, the role of taking multiple measurement decreases, and in the case of measurement error, the optimum values of the parameters are significantly affected. Prajapati and Singh [27] presented various optimal schemes of modified $\overline{X}$ chart for various sample sizes (n) at the levels of correlation (Φ) of 0.00, 0.475 and 0.95. These optimal schemes of modified $\overline{X}$ chart were compared with the Double Sampling (DS) $\overline{X}$ chart, suggested by Costa and Claro (2008). It is concluded that the modified $\overline{X}$ chart outperforms the DS chart at various levels of correlation (Φ) and shifts in the process mean.

Sgroi et al. [28] evaluated the economic and financial sustainability of lemon production, both in organic farming and in conventional farming; the two systems. Economic analysis was carried out in a representative case study located in the Sicilian northwestern coast, considering an orchard economic life equal to 50 years. They found that greater profitability of organic farming and use of environmentally friendly inputs in production process make farms competitive and eco-friendly.

2 Formulation of autocorrelated series of observations

It is assumed that each individual observation of an autocorrelated series is dependent upon the previous observation. A series of positively autocorrelated numbers with a mean of zero and standard deviation of one is generated, using the MATLAB 6.5 at various levels of correlation (Φ). If N pairs of observations on two variables x and y are assumed then the correlation coefficient (r) between x and

y is given by equation (1); as suggested by Chatfield [29]

$$r = \sum(x_i - \overline{x})(y_i - \overline{y}) / \left[\sum(x_i - \overline{x})^2\right]^{1/2} \left[\sum(y_i - \overline{y})^2\right]^{1/2}, \quad (1)$$

where, the summations are over the 'N' observations. A similar idea can be applied to a series for which successive observations are correlated. Instead of two different time series, the correlation is computed between one series and the same series lagged by one or more time units.

3 Theory of Shewhart and modified $\overline{X}$ charts

In the Shewhart $\overline{X}$ chart, only two control limits have been used to arrive at a decision to decide the control of the state of the process. In the modified $\overline{X}$ chart, two more limits at 'K' times sample standard deviation on both sides from center line have been introduced. These limits are known as warning limits. The control limits in modified $\overline{X}$ chart are also assumed at 'L' times sample standard deviation on both sides from center line. ARL of $\overline{X}$ chart depends upon selection of parameters, K and L.

For sample size of 'n', the parameters of the proposed chart: history (H), L, K and standard value of chi-square (U^*) can be selected to keep the false alarms (in-control ARL) to specific value. Both the charts are designed for the in-control ARLs of 370, so values of parameters: L, K, H and U^* are selected in such a way so that the in-control ARL of 370 for both the charts can be maintained. When any value of sample mean ($\overline{X}$) falls beyond the upper and lower control limits, it may be assumed that the process has gone out of control. But when a point falls between warning and control limits on $\overline{X}$ chart, action should not be initiated immediately, but statistic 'U' is required to be evaluated. To conclude for process to be under control with certain degree of freedom, it is required that 'U' should not exceed some predefined value (tabulated value of chi-square). In this case, for a sample size of 4 and for history (H) of 4 preceding samples, 'U' can be evaluated as given in equation (2).

$$\text{`}U\text{'} = \sum_{i=1}^{H}\sum_{j=1}^{n}((x_{ij} - \mu_0)/\sigma_0))^2, \quad (2)$$

where x_{ij} = individual measurement of jth observation of sample; i and μ_0 and σ_0 = target mean and standard deviation.

The maximum value of chi-square distribution with 16 degrees of freedom and confidence level of 95%, $U^* = 26.3$. If 'U' exceeds 26.3 with 95% confidence level, it cannot be guaranteed that the process actually has a mean of μ_0 and standard deviation of σ_0. Thus the process may be assumed out of control. The detailed procedure is shown in Singh and Prajapati [5].

4 Economic designs of modified and Shewhart $\overline{X}$ charts

The economic design of modified and Shewhart $\overline{X}$ charts are presented in this section.

4.1 Assumptions of economic designs of modified and Shewhart $\overline{X}$ charts

Following assumptions are considered in economic designs of both the charts:

(i) It is assumed that in the Shewhart $\overline{X}$ chart, only two control limits are used to arrive at a decision while in modified $\overline{X}$ chart, in addition to control limits, two more limits at 'K' times sample standard deviation are introduced.
(ii) The economic designs of control chart models assume that the process begins in a state of statistical control.
(iii) The occurrence of an assignable cause results in a shift in a process mean or dispersion.
(iv) Single/multiple assignable cause(s) may be responsible for shift in mean of the process.
(v) The time between occurrences of an assignable cause is assumed to follow an exponential distribution with a mean of λ occurrences per hour (so $1/\lambda$ is the mean time in the in-control state).
(vi) The process continues to run during the investigation of a possible special cause.
(vii) If a point falls outside of control limits, the process is assumed to be out of control and search for assignable cause is initiated. If production continuous during search of assignable causes, the expected time until an occurrence of an assignable cause is $1/\lambda$. However if production ceases during false alarm searches then we must include time spent during search process.

4.2 Lorenzen-Vance (1986) [1] Cost model

Lorenzen-Vance [1] considered a general process model that applied to all control charts regardless of statistic used.

When the assignable cause occurs, the process mean shifts $\delta\sigma$ units, then the average number of sampling before the detection of process mean shift is ARL ($\delta \neq 0$). Assuming that the sampling interval is h, the average time of occurrence within an interval between the jth sampling and $(j+1)$st sampling, given an occurrence of the shift in the interval between these samplings, is

$$\tau = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})}.$$

The out-of-control period will be: $h \times \text{ARL}\,(\delta \neq 0) - \tau$.

If production continuous during search of assignable causes, the expected time until an occurrence of an assignable cause is $1/\lambda$. However if production ceases during false alarm searches then time spent during search process is included.

The formulation of Lorenzen-Vance cost model is presented in Appendix A.

4.3 Methodology to determine the sampling interval (h) for modified $\overline{X}$ chart

Step 1 Decide the sample size (n), history (H), limiting value of chi-square distribution for 'v' degrees of freedom (U^*) at the determined level of confidence.

Step 2 Determine the level of correlation (Φ) for which average run lengths (ARLs) are to be computed.

Step 3 Find the out-of-control average run length (ARL_1) for a specific shift in the process average at the predetermined parameters for the specified in-control average run length (ARL_0) at each level of correlation.

Step 4 Decide the values of the parameters: λ, C_0, C_1, a, b, Y, W, γ_1, γ_2, T_0, T_1, T_2 and E, used in equation (A.1) (Appendix A).

Step 5 Decide the range of values of sampling interval for given set of parameters using Lorenzen and Vance cost model, used in equation (A.1) (Appendix A).

Step 6 Compute the expected cost per hour for a given set of parameters as given in equation (A.1) (Appendix A).

Step 7 Check the minimum value of expected cost per hour for a given set of parameters.

Calculations for sampling intervals and expected cost per hour for both sets of parameters are explained in the following section.

4.4 Optimal sampling interval (h) for modified $\overline{X}$ chart

In this paper, the unified cost model; developed by Lorenzen and Vance [1] is used to find the optimal sampling interval of modified $\overline{X}$ chart for sample sizes of 2 and 4. For a particular sample size, the expected costs per hour are computed for different shifts in the process mean at various levels of correlation. Optimal sampling intervals are determined for two sets of parameters; suggested by Montgomery [2], using Lorenzen and Vance [1] cost model. Sampling intervals depends upon the sample size (n); shift in process average (δ) and in-control average run length (ARL_0) along with other chart parameters. Sampling intervals are computed for sample sizes of 2 and 4 at the in-control ARL of 370, for shifts in process average of 0.5, 1.0, 1.5, 2.0 and 3.0. Two sets of parameters, suggested in Montgomery [2] are used to determine the optimal sampling intervals (h); as given below.

(i) First set of parameters

$$\lambda = 0.01,\ C_0 = 10.0,\ C_1 = 100.0,\ a = 0.5,\\ b = 0.1,\ Y = 50.0,\ W = 25.0,\ \gamma_1 = 1,\ \gamma_2 = 1,\\ T_0 = 0,\ T_1 = 2,\ T_2 = 2\ \text{ and }\ E = 0.05.$$

(ii) Second set of parameters

$$\lambda = 0.05,\ C_0 = 10.0,\ C_1 = 100.0,\ a = 0.5,\\ b = 1.0,\ Y = 50.0,\ W = 25.0,\ \gamma_1 = 1,\ \gamma_2 = 1,\\ T_0 = 0,\ T_1 = 2,\ T_2 = 2 \text{ and } E = 0.05.$$

4.4.1 Sampling intervals for modified $\overline{X}$ chart for sample size of two at various levels of correlation, using first and second set of parameters

In-control ARL of 370 is maintained for modified $\overline{X}$ chart to compute expected cost (EC) per hour for sample size of two. Both sets of parameters suggested in Montgomery [2] have been considered to find optimal sampling intervals for modified $\overline{X}$ chart. Expected costs per hour are calculated for shifts in process average of 0.5, 1.0, 1.5, 2.0 and 3.0.

Table B.1 (Appendix B) shows the expected costs/hour for modified $\overline{X}$ chart for sample size of two at zero level of correlation and for process shifts of 0.00 of 0.5, 1.0, 1.5, 2.0 and 3.0; using second set of parameters.

Table B.2 (Appendix B) shows the expected costs/hour for modified $\overline{X}$ chart for sample size of two at zero level of correlation and for process shifts of 0.00 of 0.5, 1.0, 1.5, 2.0 and 3.0; using second set of parameters.

Table 3B (Appendix B) shows the expected costs/hour for modified $\overline{X}$ chart for sample size of two at the level of correlation of 1.00 for process shifts of 0.5, 1.0, 1.5, 2.0; using second set of parameters.

Similarly, the sampling intervals for modified $\overline{X}$ chart for sample size of two at the level of correlation (Φ) of 0.50 are calculated; using both sets of parameters for different shifts in the process mean. These tables have not been included in this paper because they will not add much value in the paper except increasing the length of the paper.

4.4.2 Sampling intervals for modified $\overline{X}$ chart for sample size of four at various levels of correlation using first and second set of parameters

The sampling intervals and expected costs/hour for modified $\overline{X}$ chart for sample size of four at the levels of correlation of 0.00, 0.50 and 1.00 and process shifts of 0.5, 1.0, 1.5, 2.0 and 3.0 are computed in the same manner as calculates for sample size of 2 for both the sets of parameters. Following facts are summarized from Tables B.1, B.2, 3B and Section 4.5.1.

I. The in-control ARL of 370 is maintained for modified $\overline{X}$ chart for sample sizes of two and four at the levels of correlation (Φ) of 0.00, 0.50 and 1.00 for both sets of parameters.

II. For each level of correlation and shift in the process mean, the minimum expected cost/hour is selected as the optimal expected cost/hour.

III. Corresponding to the optimal expected cost/hour, the optimal sampling interval is obtained. The optimal sampling intervals are selected for modified $\overline{X}$ chart

Table 1. Comparison of the expected costs/hour of the modified $\overline{X}$ chart with the Shewhart $\overline{X}$ chart for sample size of two at $\Phi = 0.00$ and 0.50.

Shift in mean	Sampling Interval (h)	Level of correlation $(\Phi) = 0.00$		Level of correlation $(\Phi) = 0.50$	
		Shewhart $\overline{X}$ chart	Modified $\overline{X}$ chart	Shewhart $\overline{X}$ chart	Modified $\overline{X}$ chart
		Expected Cost/hour (EC)	Expected Cost/hour (EC)	Expected Cost/hour (EC)	Expected Cost/hour (EC)
0.00	0.1	61.09	61.09	61.09	61.09
0.50	0.1	41.00	38.93	42.57	40.20
1.00	0.1	29.60	29.07	29.93	29.48
1.50	0.1	25.61	25.65	26.39	26.05
2.00	0.1	24.68	24.60	25.09	25.03
3.00	0.1	24.31	24.29	24.35	24.33

for both the sample sizes for all the shifts in the process mean at the levels of correlation (Φ) of 0.00, 0.50 and 1.00.

IV. For a particular shift in the process mean; as the level of correlation increases, the optimal expected cost/hour also increases. This is due to the fact that the out-of-control ARL increases corresponding to the increase in the level of correlation.

V. For each level of correlation, the optimal expected cost/hour; using second set of parameters is lower than the optimal expected cost/hour for first set of parameters.

Thus in order to monitor the process economically, the second set of parameters is suggested.

The economic design of modified $\overline{X}$ chart is compared with the economic design of Shewhart $\overline{X}$ chart in the following section.

4.5 Comparison of economic design of the modified $\overline{X}$ chart with Shewhart $\overline{X}$ chart

The economic design of modified $\overline{X}$ chart is compared with the Shewhart $\overline{X}$ chart in terms of the expected cost/hour expended in implementing the charts in the industry. Both the charts are compared for sample size of 2 and 4, using second set of parameters of the economic design model.

4.5.1 Comparison of the expected cost/hour of the modified $\overline{X}$ chart with the Shewhart $\overline{X}$ chart for sample size of two

Table 1 shows the comparison of the Expected Cost/hour of the modified $\overline{X}$ chart with the Shewhart $\overline{X}$ chart for sample size of two at level of correlation (Φ) of 0.00 and 0.50.

Table 2 shows the comparison of the expected costs/hour of the modified $\overline{X}$ chart with the Shewhart $\overline{X}$ chart for sample size of two at the level of correlation of 1.00.

Table 2. Comparison of the expected costs/hour of the modified $\overline{X}$ chart with the Shewhart $\overline{X}$ chart for sample size of two at $\Phi = 1.00$.

Shift in mean	Sampling Interval (h)	Level of correlation $(\Phi) = 1.00$	
		Shewhart $\overline{X}$ chart	Modified $\overline{X}$ chart
		Expected Cost/hour (EC)	Expected Cost/hour (EC)
0.00	0.1	61.09	61.09
0.50	0.1	52.62	42.20
1.00	0.1	33.87	33.57
1.50	0.1	28.71	28.36
2.00	0.1	26.39	26.31
3.00	0.1	24.77	24.75

Tables 1 and 2 show the following facts:

I. When there is no shift in the process mean, the expected costs/hour of the modified $\overline{X}$ chart are equal to the Shewhart $\overline{X}$ chart. This is due to the fact that the in-control ARL of both the charts is maintained at 370.

II. At each level of correlation and shift in the process mean, the expected costs/hour of the modified $\overline{X}$ chart are lower than the Shewhart $\overline{X}$ chart.

III. At the highest level of correlation of 1.00 and at 0.5 σ shift in the process mean; the expected cost/hour of the modified $\overline{X}$ chart is 42.20 whereas it increases to 52.62 for the Shewhart $\overline{X}$ chart.

Thus, it is economical to use the modified $\overline{X}$ chart for the highly correlated data.

4.5.2 Comparison of the expected cost/hour of the modified $\overline{X}$ chart with the Shewhart $\overline{X}$ chart for sample size of four

Table 3 shows the comparison of the Expected Cost/hour of the modified $\overline{X}$ chart with the Shewhart $\overline{X}$ chart for sample size of four at level of correlation (Φ) of 0.00 and 0.50.

Table 4 shows the comparison of the expected costs/hour of the modified $\overline{X}$ chart with the Shewhart

Table 3. Comparison of the expected costs/hour of the modified $\overline{X}$ chart with the Shewhart $\overline{X}$ chart for sample size of four at $\Phi = 0.00$ and 0.50.

Shift in mean	Sampling Interval (h)	Level of correlation (Φ) = 0.00		Level of correlation (Φ) = 0.50	
		Shewhart $\overline{X}$ chart	Modified $\overline{X}$ chart	Shewhart $\overline{X}$ chart	Modified $\overline{X}$ chart
		Expected Cost/hour (EC)	Expected Cost/hour (EC)	Expected Cost/hour (EC)	Expected Cost/hour (EC)
0.00	0.1	61.19	61.19	61.19	61.19
0.50	0.1	38.23	37.90	43.02	39.77
1.00	0.1	25.67	25.15	29.30	28.53
1.50	0.1	25.37	25.22	26.16	25.43
2.00	0.1	24.76	24.53	25.00	24.83
3.00	0.1	24.35	24.29	24.36	24.36

Table 4. Comparison of the expected costs/hour of the modified $\overline{X}$ chart with the Shewhart $\overline{X}$ chart for sample size of four at $\Phi = 1.00$.

Shift in mean	Sampling Interval (h)	Level of correlation (Φ) = 1.00	
		Shewhart $\overline{X}$ chart	Modified $\overline{X}$ chart
		Expected cost/hour (EC)	Expected cost/hour (EC)
0.00	0.1	61.19	61.19
0.50	0.1	78.60	40.65
1.00	0.1	62.70	31.50
1.50	0.1	58.03	27.55
2.00	0.1	56.01	25.67
3.00	0.1	54.63	24.59

$\overline{X}$ chart for sample size of four at the level of correlation of 1.00.

From Tables 3 and 4, it is found that for each level of correlation and shift in the process mean, the expected cost/hour of the modified $\overline{X}$ chart is lower than the Shewhart $\overline{X}$ chart for sample size of 4 also. Thus it is economical to use the modified $\overline{X}$ chart for sample size of four for the highly correlated data.

5 Conclusions

The Lorenzen and Vance [1] model is used to formulate the economic design of the modified $\overline{X}$ chart for sample sizes of two and four. The economic performance of the modified $\overline{X}$ chart is measured in terms of the expected cost (EC) per hour for the two set of parameters. The expected cost (EC) per hour is calculated for the shifts in the process mean. It was observed that the modified $\overline{X}$ chart for sample size of two and four with second set of parameters is most economical to use in the industry.

Moreover the expected cost (EC) per hour with second set of parameters is calculated for the shifts in the process mean of the Shewhart $\overline{X}$ chart and compared with the modified $\overline{X}$ chart for sample sizes of two and four. It is observed that for a particular level of correlation, the modified $\overline{X}$ chart for sample size of four with second set of parameters is the most economical. In this paper, it is assumed that the process being monitored will follow normal distribution over time. The observations may also be assumed non-normal and economic performance of modified chart may be studied.

Only single assignable cause that is responsible for shift in process mean in this research paper is assumed, but some other researchers have considered multiple assignable causes, responsible for shift in process mean/dispersion. Therefore, the economic design of modified $\overline{X}$ chart may be extended by considering multiple assignable causes.

Appendix A

A.1 Formulation of Lorenzen-Vance cost model

The unified cost model; developed by Lorenzen and Vance [1] is used to formulate the economic designs. They defined the expected hourly production cost as the ratio of the expected cycle cost to the expected cycle time. Under the renewal reward assumption, the expected cost per hour is:

$$\begin{aligned} C &= \{E[\text{Production Cost}] + E[\text{Alarm \& Repair cost}] \\ &\quad + E[\text{Sampling cost}]\}/E[\text{Cycle Time}] \\ &= \{C_0/\lambda + C_1[-\tau + nE + h(\text{ARL}_1 + \gamma_1 T_1 + \gamma_2 T_2] \\ &\quad + (sY/\text{ARL}_0) + W + (a + bn)/h[(1/\lambda) - \tau + nE \\ &\quad + h(\text{ARL}_1) + \gamma_1 T_1 + \gamma_2 T_2]\} \\ &\quad \div \{[(1/\lambda) + [(1 - \gamma_1 s T_0/\text{ARL}_0] - \tau + nE \\ &\quad + h(\text{ARL}_1) + T_1 + T_2\} \end{aligned} \tag{A.1}$$

where the design parameters are:

$n =$ sample size
$h =$ interval between samples (sampling interval)
$\lambda =$ exponential parameter ($1/\lambda$ is the mean time to the occurrence of an assignable cause)
$C_0 =$ the cost of producing non-conformities/hour while the process is in-control
$C_1 =$ the cost of producing non-conformities/hour while the process is out of control ($> C_0$)

$ARL_0 =$ average run length when the process is in-control state
$ARL_1 =$ average run length when the process is out of control state
$h - \tau =$ the expected time between a shift in the process(assignable cause) and the next sample, where, $\tau = 1 - (1 + \lambda h)e^{-\lambda h}/\lambda(1 - e^{-\lambda h})$
$E =$ expected time to sample and chart one item
$\gamma_1 = 1$, if production continuous during searches and 0 otherwise
$\gamma_2 = 1$, if production continuous during repair of the process and 0 otherwise
$T_0 =$ expected time to search a false alarm,
$T_1 =$ expected time to discover the assignable cause
$T_2 =$ expected time to repair the assignable cause
$s =$ $e^{-\lambda h}/(1 - e^{-\lambda h})$ = expected number of samples when process is under control
$Y =$ cost to investigate false alarm
$W =$ cost of locating and repairing an assignable cause
$\alpha =$ $1/ARL_0$ = Probability of exceeding control limits when process is in-control state
$\delta =$ Shift in the process average
$p =$ $1/ARL_1$ = probability of exceeding control limits when process is out of control state.

Appendix B

Table B.1. Expected costs per hour for modified $\overline{X}$ chart for sample size of two at the level of correlation of 0.00 using first set of parameters.

Modified $\overline{X}$ chart for sample size of two at $\Phi = 0.00$				
Sample size (n)	ARL_1	Shift (δ)	Sampling interval (h)	Expected Cost/hour (EC)
2	148.8	0.5	0.1	118.93
2	148.8	0.5	0.2	133.76
2	148.8	0.5	0.3	148.59
2	148.8	0.5	0.4	163.42
2	148.8	0.5	0.5	178.25
2	50.2	1.0	0.1	109.07
2	50.2	1.0	0.2	114.04
2	50.2	1.0	0.3	119.01
2	50.2	1.0	0.4	123.98
2	50.2	1.0	0.5	128.95
2	14.5	1.5	0.1	105.50
2	14.5	1.5	0.2	106.90
2	14.5	1.5	0.3	108.30
2	14.5	1.5	0.4	109.70
2	14.5	1.5	0.5	111.10
2	5.5	2.0	0.1	104.60
2	5.5	2.0	0.2	105.10
2	5.5	2.0	0.3	105.60
2	5.5	2.0	0.4	106.10
2	5.5	2.0	0.5	106.60
2	2.4	3.0	0.1	104.14
2	2.4	3.0	0.2	104.19
2	2.4	3.0	0.3	104.29
2	2.4	3.0	0.4	104.48
2	2.4	3.0	0.5	104.67

Table B.2. Expected costs per hour for modified $\overline{X}$ chart for sample size of two at the level of correlation of 0.00 using second set of parameters.

Modified $\overline{X}$ chart for sample size of two at $\Phi = 0.00$				
Sample size (n)	ARL_1	Shift (δ)	Sampling interval (h)	Expected Cost/hour (EC)
2	148.8	0.5	0.1	38.93
2	148.8	0.5	0.2	53.76
2	148.8	0.5	0.3	68.59
2	148.8	0.5	0.4	83.42
2	148.8	0.5	0.5	98.25
2	50.2	1.0	0.1	29.07
2	50.2	1.0	0.2	34.04
2	50.2	1.0	0.3	39.01
2	50.2	1.0	0.4	43.98
2	50.2	1.0	0.5	48.95
2	14.5	1.5	0.1	25.65
2	14.5	1.5	0.2	27.05
2	14.5	1.5	0.3	28.45
2	14.5	1.5	0.4	29.85
2	14.5	1.5	0.5	31.25
2	5.5	2.0	0.1	24.60
2	5.5	2.0	0.2	25.10
2	5.5	2.0	0.3	25.60
2	5.5	2.0	0.4	26.10
2	5.5	2.0	0.5	26.60
2	2.4	3.0	0.1	24.29
2	2.4	3.0	0.2	24.48
2	2.4	3.0	0.3	24.67
2	2.4	3.0	0.4	24.86
2	2.4	3.0	0.5	25.05

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