

# Absolute optical frequency measurements of iodine-stabilized He-Ne laser at 633 nm by using a femtosecond laser frequency comb

B. Samoudi\*, M<sup>a</sup> Mar Pérez, S. Ferreira-Barragáns, and E. Prieto

Spanish Centre of Metrology, Length Area, C/ del Alfar 2, Tres Cantos 28760 Madrid, Spain

Received: 5 May 2012 / Accepted: 15 June 2012

**Abstract.** The optical frequency comb generator (OFCG) is an attractive optical reference source for various applications such as optical frequency metrology, precision spectroscopy and telecommunications [D.J. Jones, S.A. Diddams, J.K. Ranka, A. Stentz, R.S. Windeler, S.T. Cundiff, *Science* **288**, 635–639 (2000); T. Udem, R. Holzwarth, T.W. Hänsch, *Nature* **416**, 233–237 (2002); T.W. Hänsch, J. Alnis, P. Fendel, M. Fischer, C. Gohle, M. Herrmann, R. Holzwarth, N. Kolachevsky, Th. Udem, M. Zimmermann, *Philos. Trans. R. Soc. A* **363**, 2155–2163 (2005)]. In particular, the OFCG can be used as source for absolute frequency measurement, providing a precise ruler for length metrology. In the present work we describe the results of absolute frequency measurements of primary wavelength standards at 633 nm on the sixth components, d, e, f, g, h and i of the R(127) 11-5 hyperfine transition of the  $^{127}\text{I}_2$  molecule, at the Spanish Centre of Metrology, CEM. The values obtained with a femtosecond frequency comb (FC1500, Menlo Systems) at CEM are compared with the values recommended by the Consultative Committee for Length (CCL) [T.J. Quinn, *Metrologia* **40**, 103–133 (2003)]. This determination was made by beat frequency method between a femtosecond laser comb and an iodine-stabilized He-Ne laser. The difference between the mean frequency of the sixth components of the standard laser and those of CCL recommended values for the same components was found to be 6.557 kHz.

**Keywords:** Frequency combs; length standard; mode-locked laser; optical frequency metrology

## 1 Introduction

The development of the optical frequency comb generator (OFCG) has received much attention in recent years due to their enormous potential in optical frequency metrology applications. Among these, the most prominent are inter-comparison of optical frequency standards [5], ultra-high resolution spectroscopy [6], wavelength multiplexed optical communication systems, multicolour interferometric length measurements [7] as well as astronomical observations [8]. In addition, the OFCG has led to a dramatic simplification of the absolute measurement of optical frequencies, by providing the necessary division of optical frequencies at hundreds of terahertz to radiofrequency and microwave standards at the gigahertz frequency [2].

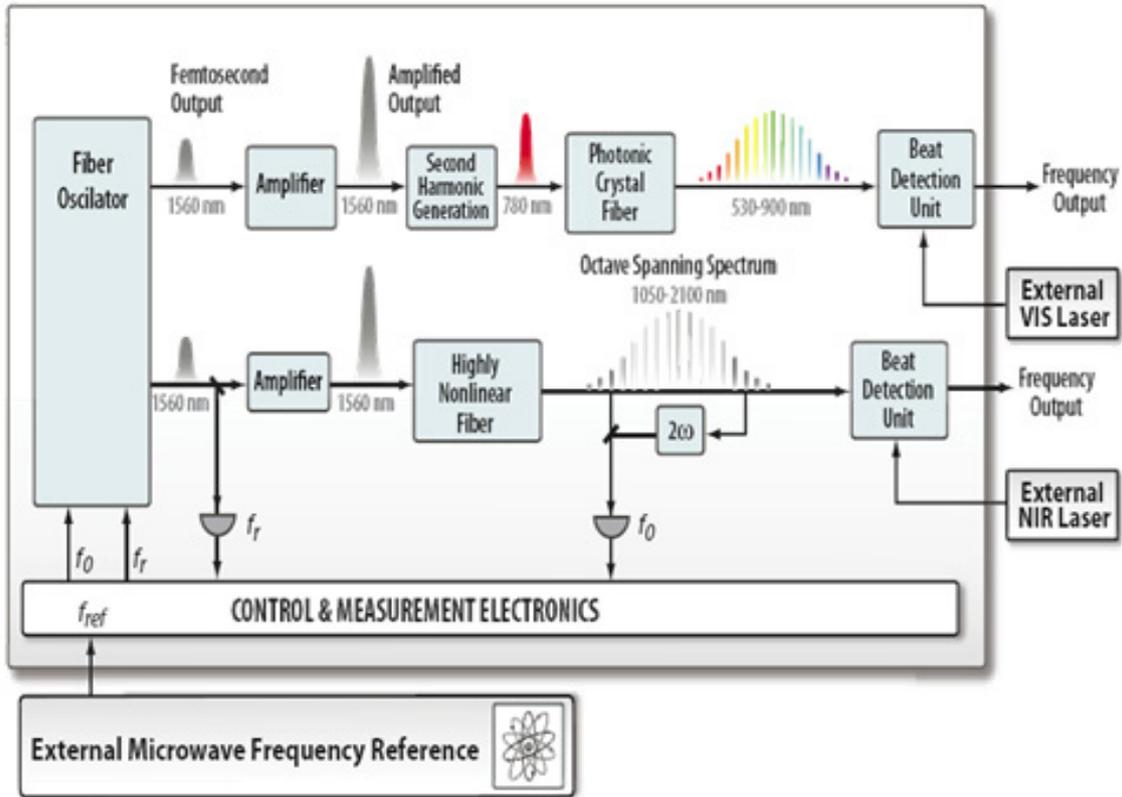
In metrology, the definition of the unit of length and its practical realisation are based on both the adopted value for the speed of light in vacuum,  $c = 299\,792\,458\text{ m s}^{-1}$ , and the frequency of a list of optical frequency standards recommended by the International Committee for Weights and Measures, CIPM, which have been updated over time [4]. Thus, length measurements are intrinsically related to the unit of time, the second.

Among the listed optical frequency standards recommended by CIPM, molecular iodine ( $\text{I}_2$ ) holds a unique position in that it offers five reference lines that have been most widely used for metrological calibration [9]. The system at 633 nm has the advantage of being one of the better quality standards based on  $\text{I}_2$ .

The adopted frequency value for the f component of the iodine-stabilized He-Ne laser of the R(127) 11-5 hyperfine transition of the  $^{127}\text{I}_2$  molecule is  $f_f = (473\,612\,353\,604 \pm 10)\text{ kHz}$ , corresponding to a relative standard uncertainty of  $2.1 \times 10^{-11}$ , for the standard laser operating conditions listed in [4]. Using an OFCG system, the Spanish Centre of Metrology has obtained preliminary results for establishing a new practical realization of the metre with an improved accuracy in three orders of magnitude with respect to the current system based on iodine stabilized lasers [10].

The details on the characterization of the frequency comb generator were published elsewhere [10]; here a brief summary was presented. A frequency comb emitted by such a device is fully determined by the laser repetition rate,  $f_r$ , which represents the spacing of the comb lines, and the carrier envelope-offset,  $f_{\text{CEO}}$ , which defines the combs offset from zero. As both  $f_r$  and  $f_{\text{CEO}}$  are in radio

\* Correspondence: [bsamoudi@cem.minetur.es](mailto:bsamoudi@cem.minetur.es)



**Fig. 1.** Experimental setup used to measure the absolute frequency of the iodine-stabilized He-Ne laser at 633 nm with a femtosecond comb generator.

frequency regime, they may be detected and counted by using standard electronics. Therefore, the OCFG modes are expressed as

$$f_n = n f_r + f_{\text{CEO}} \quad (1)$$

with a large ( $\approx 10^6$ ) integer  $n$ . This equation maps two radiofrequencies  $f_r$  and  $f_{\text{CEO}}$  onto the optical frequencies  $f_n$ .

The aim of this paper is apply the OFCG technique to measure the absolute frequency of an iodine-stabilized He-Ne laser at 633 nm nominal wavelength stabilized on the sixth components, d, e, f, g, h and i of the hyper-fine transition R(127) 11-5. The obtained result was compared with the same values recommended by the Consultative Committee for Length (CCL). The stability of both iodine-stabilized He-Ne and OFCG are reported.

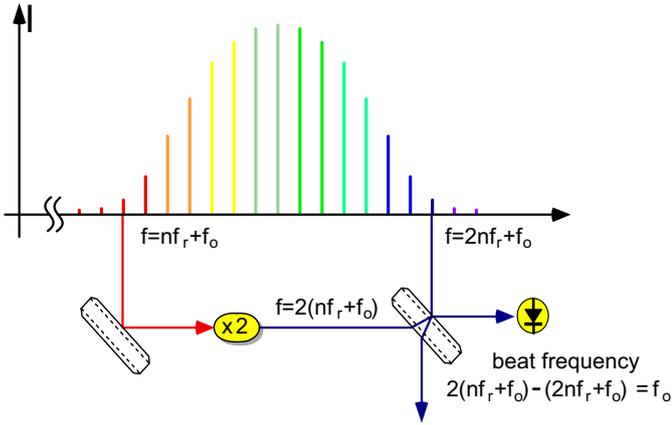
## 2 Experimental arrangements

The optical frequency comb generator (OFCG), (Menlo Systems, model FC1500) is based on doped-erbium, polarization-mode-locked, femtosecond fibre-ring laser. The OFCG offers reference frequencies with comb modes of 250 MHz spacing. The repetition rate frequency (250 MHz),  $f_r$ , and the carrier-envelope offset frequency (20 MHz),  $f_{\text{CEO}}$ , were referenced to a primary frequency standard, a Cs atomic clock (Symmetricom, model

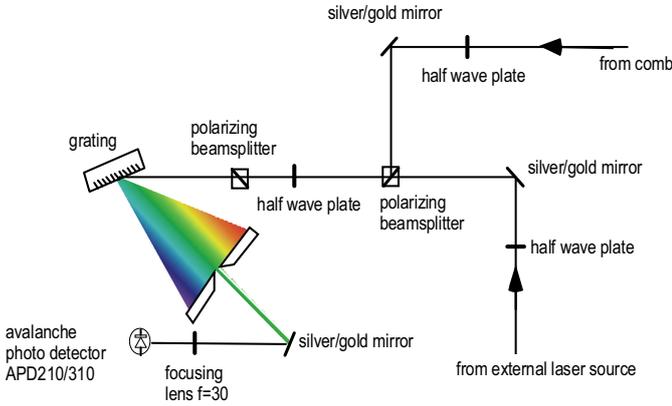
5071A), integrated into the network of atomic clocks of the Navy Observatory, ROA, in charge of maintaining the Spanish time and frequency standards. Although the detection of  $f_r$  is rather effortless, the determination of  $f_{\text{CEO}}$  is far more challenging. The experimental arrangement employed is sketched in Figure 1. The fibre oscillator consists of the laser head with internal doped fibre amplifier EDFA centred at 1500 nm with power up to 2 mW. The femtosecond laser output power is split into two branches and fed to the monitor port and the external parts of the system. One branch is amplified in an external EDFA and spectrally broadened in a highly nonlinear fibre (HNLF) to cover a spectrum of one octave in frequency space.  $f_{\text{CEO}}$  then is detected by the self-referencing technique with an  $f - 2f$  interferometer setup [1,6], by taking the difference of the frequencies of a comb mode at  $f_n$  and the second harmonic of the mode at  $f_{2n}$  (Fig. 2). The other branch for generating high power IR light at 1560 nm for subsequent frequency doubling at 780 nm radiation, and a photonic crystal fibre setup for subsequent broadening of the second harmonic generation (SHG) output to (530–1000) nm.

The beat detection unit (Fig. 3) consists of a series of mirrors coated with silver/gold, polarized beam splitters,  $\lambda/2$  waveplates, which aim to place the beams from both the laser comb and calibration in the same polarisation plane, and lead them to a photodetector.

For measuring the beat note between the iodine-stabilized He-Ne laser emitting in a nominal wavelength



**Fig. 2.** The offset frequency is found by taking the difference of the frequencies of a comb mode at  $f_n$  and the second harmonic of the mode at  $f_{2n}$ .



**Fig. 3.** Beat detection unit.

of 633 nm and de frequency comb mode, a grating (2100 lines/mm) for spectral filtering was used. Such beat note was detected by a fast photo detector APD210. All relevant frequencies in the experiment have been analyzed and counted by a spectrum analyzer (Hameg, model HM5510) and frequency counters (Menlo Systems, model FXM50), respectively, which have been referenced to a primary frequency standard (Symmetricom, model 5071A) with a relative stability of  $1 \times 10^{-12}$  at short time.

During the process of measurement, the frequency comb system as well as remaining phase is very reliably locked. The frequency counters used for this measurements count continuously every second without dead time for more than 5 h. In our case, the  $S/N$  ratio over 30 dB is sufficient for counting without cycle slips.

### 3 Absolute measurement of cw lasers

The optical frequencies of cw (continuous wave) lasers are measured by observing a beat frequency,  $f_{\text{beat}}$ , with the nearest mode of the frequency comb. Therefore, the optical frequency,  $f_{\text{cw}}$ , is expressed as

$$f_{\text{cw}} = nf_r \pm f_{\text{CEO}} \pm f_{\text{beat}}. \quad (2)$$

The mode number  $n$  is determined on the assumption that the He-Ne/I<sub>2</sub> frequency is known a priori to better than a few megahertz. The signs of the beats  $f_{\text{CEO}}$  and  $f_{\text{beat}}$  are determined by checking the increase or decrease of  $f_{\text{beat}}$  when  $f_r$  or  $f_{\text{CEO}}$  is increased under the conditions of locking  $f_r$  or  $f_{\text{CEO}}$ , respectively. If the visible cw lasers are measured after SHG, that is our case, equation (2) has to be modified to take the SHG process into consideration: the  $f_{\text{CEO}}$  has to be multiplied by a factor of two:

$$f_{\text{cw}} = nf_r \pm 2f_{\text{CEO}} \pm f_{\text{beat}}. \quad (3)$$

The repetition rate remains unchanged due to the fact that the dominant process in the SHG is sum frequency generation.

### 4 Stability of the system

The Allan deviation [ $\sigma_y(\tau)$ ] is extremely useful for characterising a frequency source because the type of phase noise present is revealed by the way in which  $\sigma_y(\tau)$  depends on the sampling time,  $\tau$ . However, for the Allan deviation to reliably indicate the type of noise present, it is crucial that there be no dead time between the consecutive average frequency measurements used to calculate  $\sigma_y(\tau)$ .

To study the stability of the system as well as the note beat we have used the Allan variance analysis [11, 12]. For a set of  $N$  frequency measurements, the Allan variance is defined as

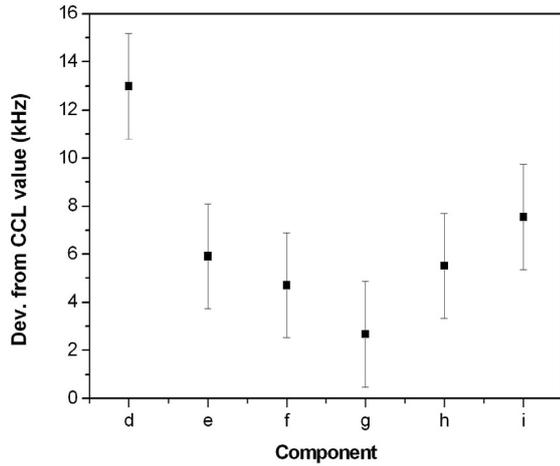
$$\sigma_y^2(\tau) = \frac{1}{2(N-1)} \sum_{i=1}^{N-1} (f_{i+1} - f_i)^2 \quad (4)$$

where  $f_i$  denotes consecutive measurements of the average frequency, averaged over a period  $\tau$ .

The Allan deviation for averaging times that are integer multiples of  $\tau$ ,  $\tau_y(m\tau)$ , can then be calculated by forming a new set of  $N/m$  average frequency values from the original set of  $N$  values. The original set is subdivided into adjacent, non overlapping subsets. Each value of the new set of frequency values is computed by averaging the  $m$  values in each subset of the original data. This is the prerequisite to be able to specify the Allan standard deviation and the standard error for the mean for integration times other than 1 s later in this article.

### 5 Results and discussion

At CEM we count with several He-Ne lasers with a nominal wavelength at 633 nm stabilized to hyperfine components of transition 11-5 R(127) of <sup>127</sup>I<sub>2</sub> vapour in internal cell with third harmonic locking technique. Here we present the results of absolute frequency measurements of laser CEM2 (Winters Electro-Optics, model WEO116) which is in operation since 1993 and which was among others compared with *Bureau International des Poids et*



**Fig. 4.** Frequencies of d, e, f, g, h and i components of the 633 nm laser measured by a femtosecond comb at CEM, printed as differences from *MeP* values. All values in kHz.

*Mesures* (BIPM) in 1994 [13] and 1995 [14]. The frequency of CEM2 was consistent with the other participating lasers in the comparison, i.e., within several kHz. In 2006 it was also measured absolutely against the BIPM Ti: sapphire-based frequency comb. The measured optical frequency,  $f_f$ , of the standard laser agreed with the CIPM recommended frequency value within 5.1 kHz. Recently, the absolute frequency of the CEM2 laser,  $f_f$ , was measured using the Bundesamt für Eich- und Vermessungswesen (BEV) fiber femtosecond laser comb set-up following the technical protocol for the key comparison CCL-K11 for optical frequency/wavelength standards. The absolute frequency uncorrected to specified operating conditions was found to be

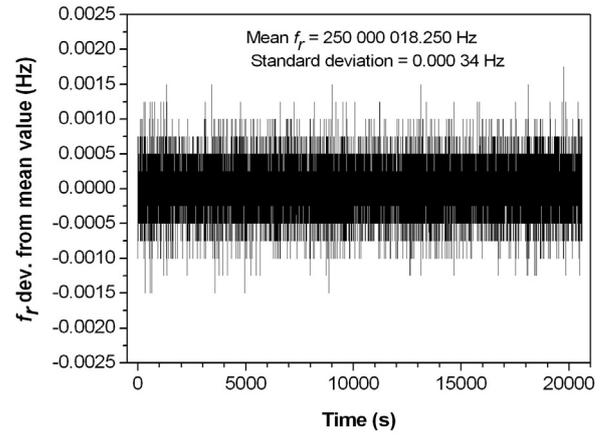
$$f_f = (473\,612\,353\,611.6 \pm 0.1) \text{ kHz.}$$

The results of measurements of the six components of laser CEM2 with the comb at CEM are in very good agreement with the BIPM ones, except the component d. In effect, the frequency of this component at recommended conditions is

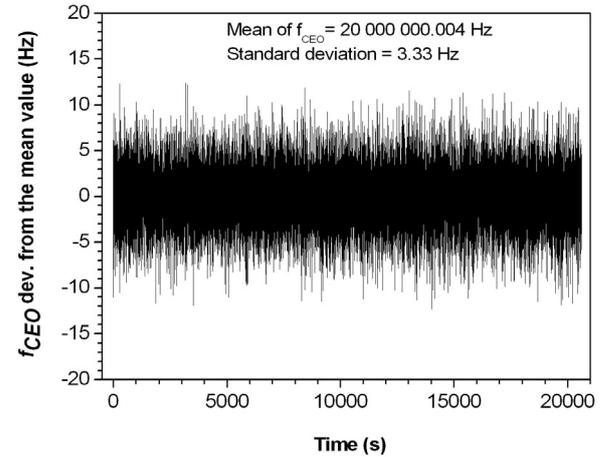
$$f_{\text{CEM2}} = (473\,612\,379\,840.983 \pm 4.378) \text{ kHz}$$

i.e. 12.983 kHz above the MeP value. This relatively high deviation (still within 20 kHz expanded uncertainty) of *MeP* may be partially due to the relative drift or degradation of the laser. Figure 4 and Table 1 show these results compared with the recommended values by the CCL. The uncertainties quoted in this work are expanded ( $k = 2$ ) and correspond to the uncertainty of the femtosecond comb combined with the repeatability of the laser under measurement.

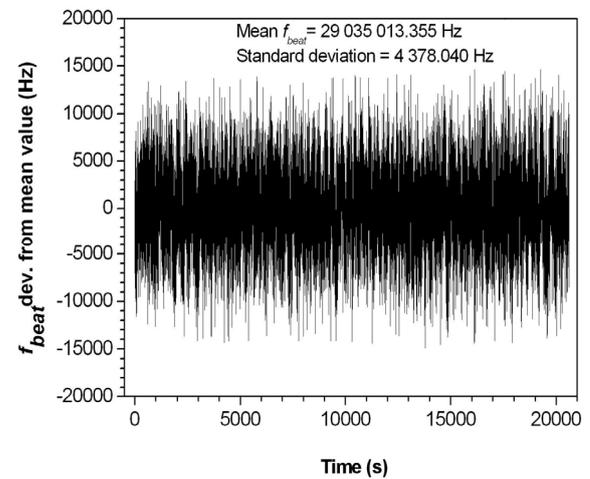
The mean  $f_r$ ,  $f_{\text{CEO}}$ ,  $f_{\text{beat}}$  (CEM2 stabilized on  $f$  component) and synthesizer frequency deviation measurements are shown in Figures 5a–5c. It can be seen that the standard deviations for these measurements are in agreement with the acceptance and rejection criteria established for obtaining reliable results.



(a)

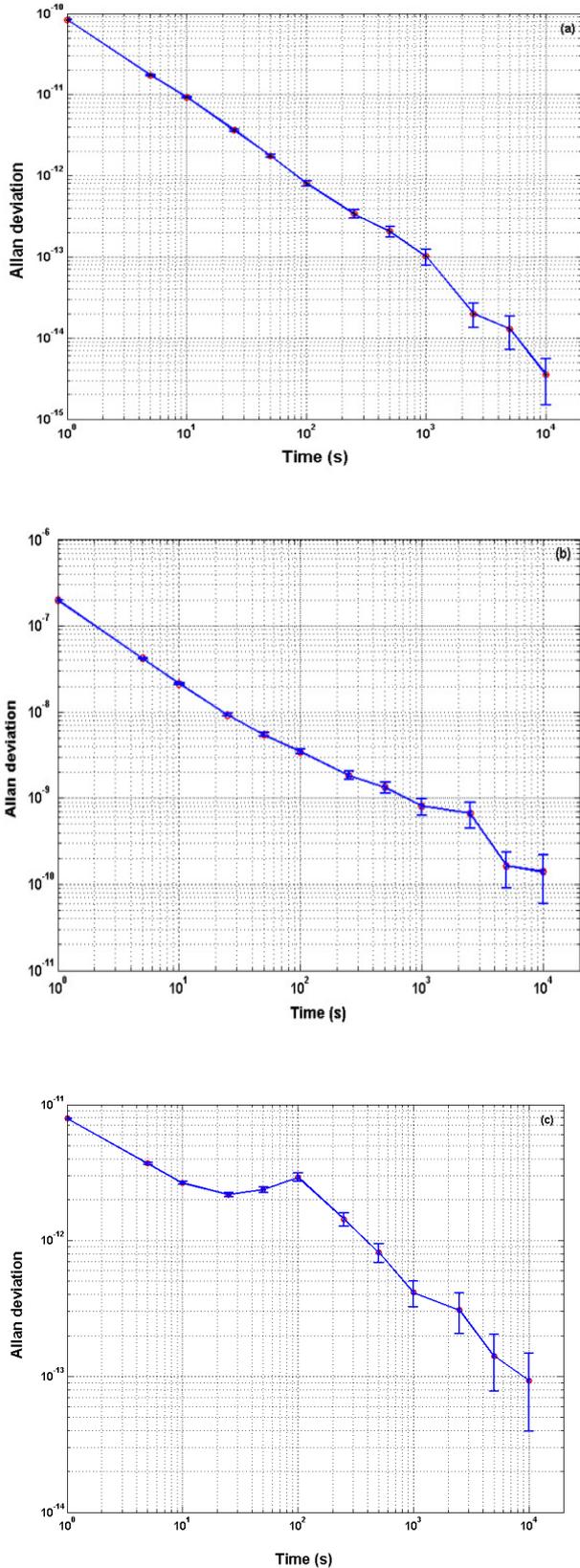


(b)



(c)

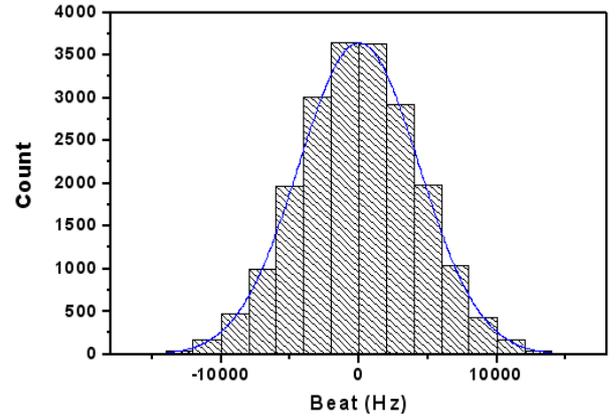
**Fig. 5.** (a) Repetition rate frequency shows a mean frequency of 250 000 018.250 Hz and a standard deviation of 0.000 34 Hz. (b) Carrier-envelope offset frequency shows a mean frequency of 20 000 000.004 Hz and a standard deviation of 3.33 Hz. (c) Repetition rate frequency shows a mean frequency of 29 035 013.355 Hz and a standard deviation of 4378.040 Hz.



**Fig. 6.** (a) Allan deviation of repetition rate frequency. (b) Allan deviation carrier-envelope offset frequency. (c) Allan deviation of  $f_{\text{beat}}$  between CEM2 stabilized on f component and femtosecond comb.

**Table 1.** Frequencies of d, e, f, g, h and i components of 633 nm laser measured by femtosecond comb at CEM, printed as a difference from mise en pratique (*MeP*) values. All values in kHz.

Component	<i>MeP</i> frequency	Deviation of CEM2
d	473 612 379 828	$12.983 \pm 4.378$
e	473 612 366 967	$5.914 \pm 4.378$
f	473 612 353 604	$4.708 \pm 4.378$
g	473 612 340 406	$2.671 \pm 4.378$
h	473 612 236 651	$5.517 \pm 4.378$
i	473 612 214 712	$7.549 \pm 4.378$



**Fig. 7.** Histogram of counting quality of  $f_{\text{beat}}$  of standard laser with femtosecond comb.

The stability of the measured absolute frequency of our wavelength standard is limited by the stability of the reference Cs clock and a Rb clock disciplined to the previous one, for ensuring long and short term stability.

After calculating the Allan deviation (square root of the Allan variance), the stability of the femtosecond comb reaches  $8.38 \times 10^{-11}$  at 1 s averaging and improves to  $3.54 \times 10^{-15}$  after 10 000 s. Figures 6a–6c show the stability of the global system as well as the frequency beat. It can be seen that Allan deviation of beat frequency obtained with the standard He-Ne laser against the frequency comb describes the typical behavior with white noise common in the cesium and rubidium standards.

Figure 7 shows typical histogram obtained in the measurement of a standard laser with a femtosecond comb.

## 6 Conclusion

The optical frequency comb generator at CEM was successfully put into operation. We have measured the absolute frequency of the CEM2 standard laser at 633 nm stabilized on the d, e, f, g, h and i components of the 11-5 R(127) hyperfine transition of the  $^{127}\text{I}_2$  molecule. The result of measurements was in good agreement with previous studies as well as the difference between the mean frequency of the six components of the standard laser and those of CCL recommended values for the same components were found to be 6.557 kHz. In addition the stability of beat frequency is dominated by the typical white noise.

*Acknowledgements.* Authors wish to thank the Spanish Centre of Metrology, CEM, and the Ministry of Industry, Energy and Tourism, MINETUR, for their support to this work.

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