

RESEARCH ARTICLE

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Prediction of machining errors for free-form surfaces based on an improved grey model (1,N)

Dayang Ma and Yueping Chen 

School of Mechanical and Automotive Engineering, Guangxi University of Science and Technology, Liuzhou 545616, China

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Abstract. An improved grey model (GM) for predicting free-form surface machining errors was established to address the low measuring efficiencies of coordinate measuring machines (CMMs) and the low prediction accuracy of the GM(1,1) model. A number of points on a free-form surface are measured with a CMM, and machining errors are obtained. Ideas from metabolic methods and the GM(1,N) prediction model were combined. To reduce the impact of random fluctuations of the machining errors, a Markov prediction model was then used to correct the fitted results for the residuals and obtain the predicted results of the machining errors for free-form surfaces, thus improving the prediction accuracy of the model. The predicted results of the metabolic GM(1,1) and metabolic GM(1,N) models were then compared. The experimental results showed that the combination of metabolic theory, a grey model, and Markov theory effectively improved the prediction accuracy.

Keywords: GM(1,N) / free-form surfaces / prediction of machining errors / metabolism / Markov theory

1 Introduction

Coordinate measuring technology is widely applied in industrial production [1]. A coordinate measuring machine (CMM) can accurately obtain information about the machining errors at various points on a work piece surface. Free-form parts are generally measured using high-precision CMMs. Measurement accuracy is usually improved by increasing the number of measuring points, which is both time consuming and laborious [2]. A machining error prediction model can predict machining errors at any location on the surface of a free-form work piece from machining error information at a limited number of measurement points to obtain richer machining error information. When the prediction errors are sufficiently small, the prediction model can replace the CMM to obtain information about the machining errors at unmeasured points, thus improving the inspection efficiency.

To make an effective prediction of machining errors in a free-form part [3], a machining error prediction model must first be developed. Thus, it is crucial to seek an efficient, simple, and highly accurate modeling method for machining error prediction. Scholars around the world have conducted considerable research on error prediction, mainly based on grey system theory, artificial neural network theory, and other methods. Accurate prediction of

machining errors helps shorten the production cycle time, reduce product scrap rates, and improve the effectiveness of error compensation [4–6].

Some scholars have chosen single prediction models for the prediction of machining errors and optimized the models using other mathematical methods. Qi et al. [7] established a mathematical model for the dynamic distribution of dimensional errors in the bearing grinding process based on the characteristics of mechanical machining errors using grey system theory. Analysis and prediction of the dynamic distribution of current and future machining errors were conducted based on the model. The experimental results showed that the accuracy of the model was high and the predicted values matched well with the measured values. Sun et al. [8] proposed a machining error prediction model for thin-walled parts and obtained data through simulations and experiments. Compared with the mechanism model or data-driven model, the experimental results showed that the proposed model has higher efficiency and accuracy. Wang [9] used a neural network model for the prediction of machining errors. Huang et al. [10] used a simulated annealing algorithm to optimize a back propagation neural network to effectively predict machining errors on free-form surfaces. Wu et al. [11] proposed a gear machining error prediction method based on a parameter significance estimation and probability regression, which was experimentally verified to be superior to other algorithms in predicting gear inspection accuracy. Manikandan et al. [12]

* Corresponding author: chenyueping99@126.com

developed a geometric error prediction model using process geometry variables, such as the actual feed per revolution, as reference factors. The model enabled more effective prediction and control of geometric errors in unmachined conditions. Yang et al. [13] studied the background values of the grey model (GM) and proposed an optimization method based on three-parameter background values, which could effectively improve the smoothing effect of the background values and weaken the influence of extreme data on the performances of grey models.

Single forecasting models are often inadequate, and many scholars have sought to improve the forecasting accuracy by combining forecasting models. Shen et al. [14] combined a grey model and a neural network model, invoking a particle swarm algorithm to optimize a grey neural network prediction model. Li et al. [15] established a dynamic analysis model by optimizing a grey model through a Taylor approximation and a Markov prediction model and verified the validity of the prediction model through experiments. Zhou et al. [16] proposed a model for predicting machining errors by combining a metabolic grey model and a non-linear autoregressive neural network for the dynamic and non-linear characteristics of machine tool spindle machining errors. The results showed that the combined model had high accuracy, speed, and robustness, it outperformed the individual models, and it could be applied for the prediction of other complex machining errors. Zhao et al. [17] implemented a GM(1,1) model for the replacement of old and new information in the prediction process and combined it with Markov chains to correct the prediction results. The method is more suitable for long-term forecasting with a large volume of data and a wide range of fluctuations, which can effectively compensate for the shortcomings of the grey model and improve the feasibility of the forecast values. Yang et al. [18] analyzed the characteristics and shortcomings of the traditional grey model, combined with the advantages of Markov chain theory, which is suitable for dealing with processes with large random fluctuations, and proposed a weighted Markov combined prediction model based on the dimensional prediction of the external turning process. Zou et al. [19] used a GM(1,1) model in grey system theory to model the predicted relative errors of 12 holes drilled and reamed consecutively and used Markov theory to classify the predictions into states and correct the results. The results of this study demonstrated that it is feasible to use a combination of grey models and Markov chain theory for the prediction of processing errors.

Lu et al. [20] analyzed the relevant factors affecting the passenger volume of urban rail transport and proposed a combined forecasting model by integrating the ideas of a GM(1,N) forecasting model, Markov theory, and three metabolic methods. Fung et al. [21] optimized two different prediction models using recursive extended least squares (RELS) and neural network methods, and the results showed that the combined prediction model was more effective. Teng et al. [22] combined metabolic theory, a grey model, and Markov theory to construct a dynamic online public opinion crisis prediction model with a metabolic GM(1,N) Markov model. Wang et al. [23] constructed a grey

incremental model and a grey combined prediction model based on the GM(1,1) model principle and used the least squares method to solve for the optimal weight coefficients of the grey combined prediction model.

In summary, although many research results have improved and applied grey models more prominently, the cross-fusion application research using metabolic theory, grey models, and Markov theory at the same time has not yet been completed. In particular, the application of these fusion methods for free-form surface machining error prediction needs to be studied in depth. Therefore, this article proposes a method for predicting free-form machining errors based on a metabolic grey Markov model.

2 Development of metabolic GM(1,N) Markov model

2.1 Modeling ideas

Based on the prediction results of the GM(1,N) model, this study combines the ideas of three approaches: grey theory, Markov theory, and metabolic methods. A new free-form surfaces machining error prediction model is proposed called the metabolic GM(1,N) Markov model. The modeling steps are as follows.

Step 1. Obtain the original data series $X_i^{(0)} = [x_i^{(0)}(1), x_i^{(0)}(2), \dots, x_i^{(0)}(n)], i = 1, 2, \dots, N$. Initialize the raw data and process it non-negatively.

Step 2. Determine the best predictive dimension N for the selected series.

Step 3. Establish the GM(1,N) model for prediction.

Step 4. Calculate the residuals. Calculate the degree of deviation from the actual values.

Step 5. Perform state partitioning and build a state transfer probability matrix, use a Markov prediction model for residual correction, and then refit the data.

Step 6. Use the new fitted data to optimize the model to obtain new predicted values.

Step 7. Use metabolic ideas for the replacement of old and new values to obtain the final prediction.

The overall modeling process is shown in Figure 1.

2.2 GM(1,N) model

Grey theory was proposed by Professor Deng Julong in 1982, and grey prediction models are core elements of grey theory. The grey model GM(1,N) transforms the original data into a series with regular variation, weakening the randomness of the original data, and the constructed first-order linear differential equation is solved by cumulative reduction to find the predicted value of the original data series. The modelling process of the GM(1,N) model is shown in Figure 2, and the steps are as follows.

The system has N sequences of data containing n elements:

$$X_i^{(0)} = [x_i^{(0)}(1), x_i^{(0)}(2), \dots, x_i^{(0)}(n)], i = 1, 2, \dots, N. \quad (1)$$

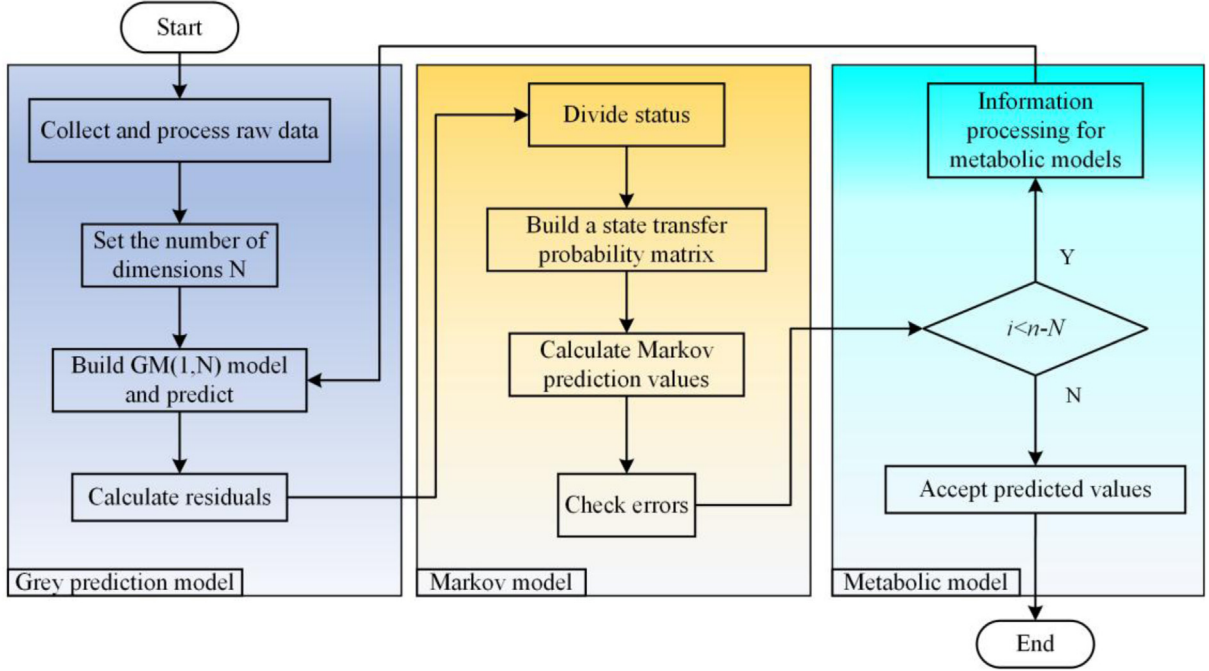


Fig. 1. Modeling process for the metabolic GM(1,N) Markov model.

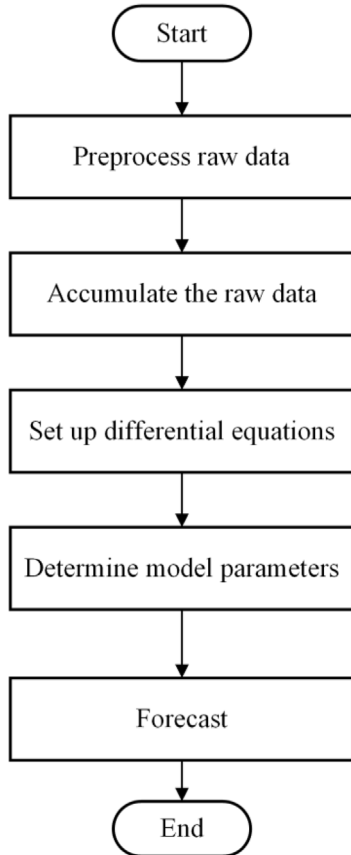


Fig. 2. Modeling process for the GM(1,N) model.

An accumulation on $X_i^{(0)}$ is performed, and the one-time accumulated generating series is

$$X_i^{(1)} = [x_i^{(1)}(1), x_i^{(1)}(2), \dots, x_i^{(1)}(n)], i = 1, 2, \dots, N, \quad (2)$$

where

$$x_i^{(1)}(k) = \sum_{i=1}^k x_i^{(0)}(i), k = 1, 2, \dots, N. \quad (3)$$

The background value $Z^{(1)}(k)$ is defined as follows:

$$Z^{(1)}(k) = \frac{1}{2} [x^{(1)}(k) + x^{(1)}(k-1)], k = 2, 3, \dots, N. \quad (4)$$

Finally, the GM(1,N) grey differential equation for $X_i^{(1)}$ is constructed:

$$X_1^{(0)}(k) + \alpha Z_1^{(0)}(k) = \sum_{i=2}^N \beta_i X_i^{(0)}(k). \quad (5)$$

The GM(1,N) whitening differential equation is

$$\frac{dX_1^{(1)}}{dt} + \alpha X_1^{(1)} = \beta_2 X_2^{(1)} + \beta_3 X_3^{(1)} + \dots + \beta_N X_N^{(1)}, \quad (6)$$

where $\alpha, \beta_1, \beta_2, \dots, \beta_N$ are parameters.

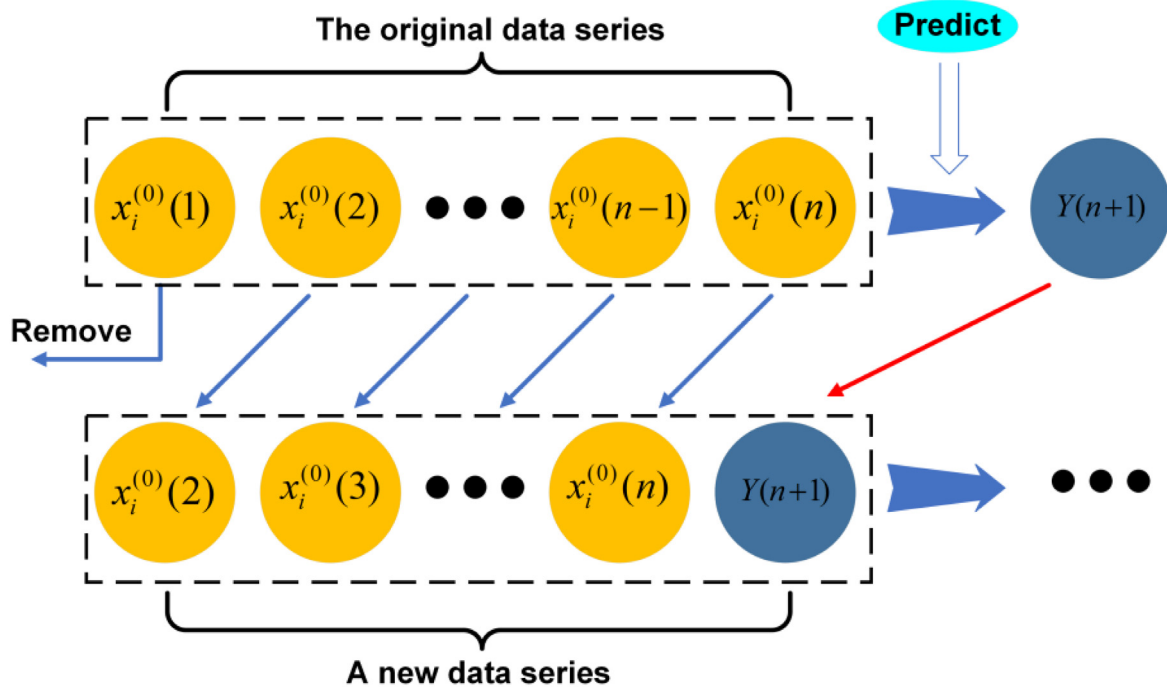


Fig. 3. Schematic diagram of the metabolic model.

The series $Y_n = [x_i^{(0)}(1), x_i^{(0)}(2), \dots, x_i^{(0)}(n)]^T$ and the parametric series $\hat{a} = (\alpha, \beta_1, \beta_2, \dots, \beta_N)^T$ are defined. Then,

$$Y_n = B\hat{a}. \quad (7)$$

From this it can be determined that

$$\hat{a} = (B^T B)^{-1} B^T Y, \quad (8)$$

$$\text{where } B = \begin{bmatrix} -z^{(1)}(2) & x_2^{(1)}(2) & \dots & x_N^{(1)}(2) \\ -z^{(1)}(3) & x_2^{(1)}(3) & \dots & x_N^{(1)}(3) \\ \vdots & \vdots & \ddots & \vdots \\ -z^{(1)}(n) & x_2^{(1)}(n) & \dots & x_N^{(1)}(n) \end{bmatrix}.$$

Parameter $\hat{a}$ is substituted into equation (3), yielding

$$\begin{aligned} \hat{x}_1^{(1)}(k+1) &= \left[x_1^{(0)}(1) - \frac{1}{\alpha} \sum_{i=2}^N \beta_i x_i^{(1)}(k+1) \right] e^{-ak} \\ &+ \frac{1}{\alpha} \sum_{i=2}^N \beta_i x_i^{(1)}(k+1), \end{aligned} \quad (9)$$

Where α and β_i are parameters.

The solution obtained by equation (9) is the predicted value of the original cumulative series, so the predicted value of the original series under the GM(1,N) model needs to be obtained by cumulative reduction:

$$\hat{x}_1^{(0)}(k) = \hat{x}_1^{(1)}(k) - \hat{x}_1^{(1)}(k-1). \quad (10)$$

$\hat{x}_1^{(0)}(k)$ is the predicted value of the original series under the GM(1,N) prediction model:

$$\varepsilon^0(k) = x_1^{(1)}(k) - \hat{x}_1^{(1)}(k). \quad (11)$$

Note that the original residuals generated are $\varepsilon^0 = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)$, where ε^0 is the sequence of residuals from $X_1^{(0)}$. The residual series are used to build a GM(1,N) model and find its parameters $P = [\alpha_\varepsilon, \beta_\varepsilon]^T$. The simulated value of $\hat{\varepsilon}^0(k+1)$ is calculated according to equation (11):

$$\hat{\varepsilon}^0(k+1) = (-\alpha_\varepsilon) \left(\varepsilon^0(k_0) - \frac{\beta_\varepsilon}{\alpha_\varepsilon} \right) e^{-\alpha_\varepsilon(k-k_0)}, \quad (12)$$

where α_ε and β_ε are parameters. Then, equation (9) can be corrected to

$$\begin{aligned} \hat{x}_1^{(1)}(k+1) &= \begin{cases} \left[x_1^{(0)}(1) - \frac{1}{\alpha} \sum_{i=2}^N \beta_i x_i^{(1)}(k+1) \right] e^{-ak} + \\ \frac{1}{\alpha} \sum_{i=2}^N \beta_i x_i^{(1)}(k+1), & k < k_0 \\ \left[x_1^{(0)}(1) - \frac{1}{\alpha} \sum_{i=2}^N \beta_i x_i^{(1)}(k+1) \right] e^{-ak} + \\ \frac{1}{\alpha} \sum_{i=2}^N \beta_i x_i^{(1)}(k+1) \pm \hat{\varepsilon}^0(k+1), & k > k_0. \end{cases} \end{aligned} \quad (13)$$

2.3 Metabolic GM(1,N) model

When using the GM(1,N) forecasting model, the data source is fixed and the model is unable to provide up-to-date data for the data used in the forecasts. Using such a fixed data source for forecasting more distant data is bound to produce large errors in the forecasts, whereas the metabolic idea can incorporate the latest forecasts already produced into the model. Therefore, the idea of metabolism is incorporated in this study by adding the predicted data as current information when predicting the value of the $n+2$ location points and then removing the information from the earliest data. The process is as follows. The original data series $X_i^{(0)} = [x_i^{(0)}(1), x_i^{(0)}(2), \dots, x_i^{(0)}(n)]$, $i = 1, 2, \dots, N$ is added to the predicted data $Y(n+1)$ as a new data source, and then the earliest data $x_i^{(0)}(1)$ is removed. As a result, a new data series $X_i^{(0)} = [x_i^{(0)}(2), x_i^{(0)}(3), \dots, x_i^{(0)}(n), Y(n+1)]$, $i = 1, 2, \dots, N$ can be obtained, which, using this data series, enables the data to be kept up to date. The schematic diagram of the metabolic model is shown in Figure 3.

2.4 Metabolic GM(1,N) Markov model

The GM(1,N) model is generally used for forms where the variation in the original series has an exponential pattern and where the dynamic fluctuations in the surface machining errors can produce large errors in the prediction results. Therefore, based on the prediction situation of the GM(1,N) model and the characteristics of the fluctuations in the free-form surface processing errors, Markov theory was used to classify the prediction results into states, and a Markov model was used to make a quadratic correction to the GM(1,N) prediction results in order to make up for the limitations of the GM(1,N) model and improve the prediction accuracy. The modelling process of the model is shown in Figure 4 and the specific modelling steps are as follows.

(1) Division of state intervals

The Markov model state partitioning method based on the GM(1,N) model is based on the relative error distribution of the prediction results of the GM(1,N) model, resulting in a corresponding scatter plot and a number of lines parallel to the x-axis. The area between each pair of adjacent parallel lines can be regarded as a state, denoted as state E_i .

The number of state intervals depends in general on the size of the original data set. As the total number of state transfers is smaller when the volume of raw data is smaller, the number of states should be smaller in order to reflect the transfer of raw data between states more objectively. In contrast, when the volume of the original data is larger, the number of states should be divided into a larger number to extract more information from the state transfer probability matrix to improve the prediction accuracy.

(2) State transfer probability matrix

The number of transfers from state E_i to state E_j is $n_{i \rightarrow j}$ and the number of transfers from state E_j to another state is N_i . The state transfer probability of transferring from state

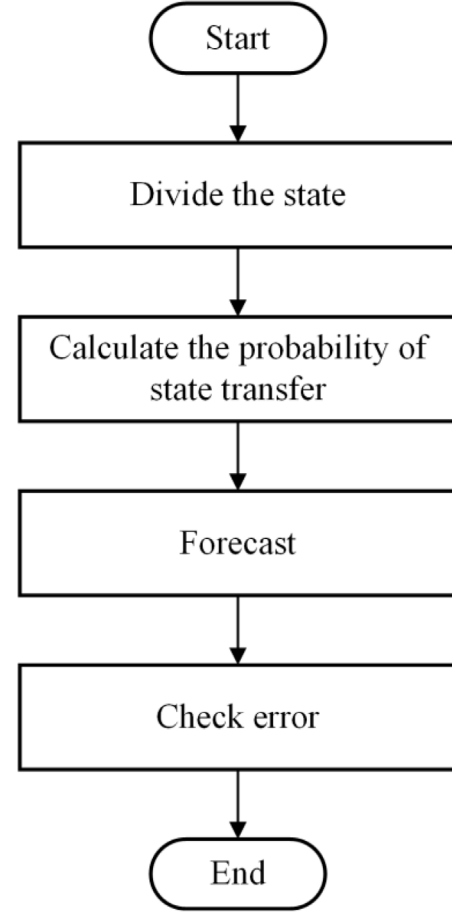


Fig. 4. Prediction process for Markov model.

E_i to state E_j is

$$P_{i \rightarrow j} = \begin{cases} \frac{n_{i \rightarrow j}}{N_i}, & N_i > 0. \\ 0, & N_i = 0 \end{cases} \quad (14)$$

According to equation (14), the state transfer probability matrix can be calculated as

$$P = \begin{bmatrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2n} \\ \vdots & \vdots & \dots & \vdots \\ P_{n1} & P_{n2} & \dots & P_{nn} \end{bmatrix}, \quad (15)$$

where $P_{n1} + P_{n2} + P_{n3} + \dots + P_{nn} = 1$, $0 \leq P_{nn} \leq 1$, $i, j = 0, 1, 2, \dots, n$.

(3) Calculation of predicted values

After establishing the state transfer probability matrix P , let the object be in state E_i at moment n . If the k th row in P satisfies $\max P_{ij} = P_{kl}$, then it is considered that moment $n+1$ is most likely to be transferred from state E_i to state E_j . Thus, the interval of change $[E_{1i}, E_{2i}]$ of the predicted value is also determined, and the median of this interval

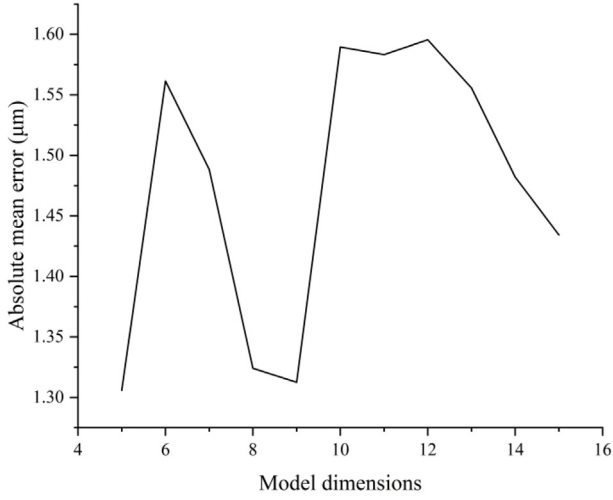


Fig. 5. Variation of prediction accuracy of metabolic GM(1,N) model with different values of dimension N .

can be taken as the predicted value at moment $n+1$:

$$\hat{y}(n+1) = \frac{1}{2}(E_{1i}, E_{2i}). \quad (16)$$

(4) Error checking

The relative error is calculated as follows:

$$\Delta\varepsilon = \frac{|\varepsilon(n)|}{x^{(0)}(n)}. \quad (17)$$

After completing the modeling of the Markov model, the metabolic model, the GM(1,N) model, and the Markov model were combined to construct the metabolic GM(1,N) Markov dynamic model.

3 Determination of the optimal dimensions of the model

From the modeling process of the metabolic model, it was found that the model dimension N was one of the main factors affecting the prediction accuracy of the metabolic model. The determination of the modeling data length (model dimension N) is very important in the metabolic modeling process, and different model dimensions N had different effects on the prediction accuracy. When the model dimension was small, the prediction accuracy of the model was low. As the number of dimensions of the model increased, the prediction accuracy of the model gradually improved. However, further increasing the number of dimensions did not improve the accuracy of the model, but tended to decrease it. There is currently no better way of determining the optimal model dimensionality. In this study, experimental calculations were performed on sample data with lengths between 5 and 15, and the mean absolute errors of the residuals of the prediction model were compared to determine the size of the selected dimension. The variation in the prediction accuracy of the metabolic

model for different dimensions N is shown in Figure 5. For this artefact model, when N was 9, the absolute mean error was minimized, and the highest model accuracy prediction precision was achieved.

The prediction accuracy of Metabolic GM(1,N) Markov model was evaluated using three sets of data when different dimensions are taken. In this paper, the mean absolute percentage error was chosen to illustrate the prediction level of the forecasting model. As can be seen in Figure 6, the prediction level of the model is changed with the choice of dimensions. The model has the best accuracy when the optimal dimension is determined to be $N=9$, so in the following tests, the model dimension is 9. The variation in the prediction accuracy of Metabolic GM(1,N) Markov model for different dimensions N is shown in Figure 6.

4 Experimental study of machining error prediction for free-form surfaces

CMMs have an unmatched advantage for dimensional inspection and can provide accurate information on machining errors at various points on a workpiece [24]. Compared with non-contact probes, touch probes have a high level of measurement accuracy. However, when measuring surface machining errors on workpieces, each point along the path has to be measured one by one, which is inefficient. A free-form surface machining error prediction model uses machining information from a limited number of known points to predict machining errors at other locations on the free-form surface. In the event that the predicted error meets the accuracy requirements, it can replace the CMM for predicting the machining error of unknown points, which achieves the purpose of improving efficiency and lowering costs.

4.1 Collecting machining error data

The workpiece used in this study was a free-form surface machined on a computer numerical control (CNC) machine. Figure 7 shows the computer-aided design (CAD) model of the free-form surface and the theoretical measurement points. The information from 300 random measurement points on the surface was selected as raw data, and a further 100 points were selected in the same way as a validation sample. The free-form surface was also inspected using a CMM to obtain the machining error of each point on the free-form surface and the corresponding coordinate values. The inspection device was a German Hexagon Leitz Reference HP CMM (PC-DMIS software, MPEE = $0.9 + L/400 \mu\text{m}$), with a selected ball diameter of 5 mm, positioning and retraction distance of 3 mm, moving speed of 20 mm/s, and touch and retraction speed of 2 mm/s. Figure 8 shows the use of a CMM for free-form machining error detection. The sample data were processed using the MATLAB R2016b platform. The selected X, Y, and Z coordinate values of the theoretical points of the free-form surface were used as influencing factors to obtain a comprehensive prediction of the machining error at each point of the workpiece.

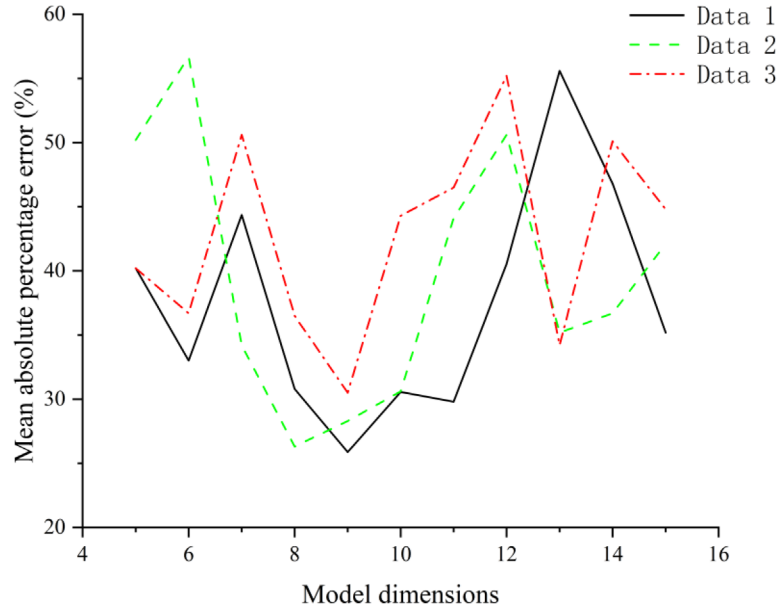


Fig. 6. Variation of prediction accuracy of Metabolic GM(1,N) Markov model with different values of dimension N .

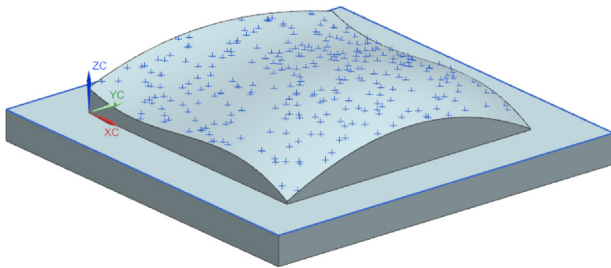


Fig. 7. Free-form surface digital model and theoretical measurement points.

4.2 Modeling and prediction of surface machining errors

Based on the machining errors obtained from the CMM inspection, a GM(1,N) model was constructed consisting of the free-form machining errors with coordinate values X , Y , and Z . Since the GM(1,N) model requires the original series to be non-negative, non-negative processing of the original data was carried out before the prediction, and the data reduction process was conducted after the prediction results were obtained.

The model accuracy was the highest when the model dimension N was taken as 9. Therefore, the model dimension was taken as 9 for modelling in the modelling process. A metabolic GM(1,N) model was used to fit the experimental data, and the results of the fit are shown in Figure 9. From the fitting results, it can be seen that a



Fig. 8. Free-form surface part measurement using the coordinate measuring machine.

better fit was obtained by using the metabolic GM(1,N) prediction model. Thus, the metabolic GM(1,N) prediction model can be used for prediction.

The metabolic GM(1,1) and GM(1,N) prediction models were used to predict the sample data, and the prediction results are shown in Figure 10. It is clear that the metabolic GM(1,N) model had a better prediction accuracy.

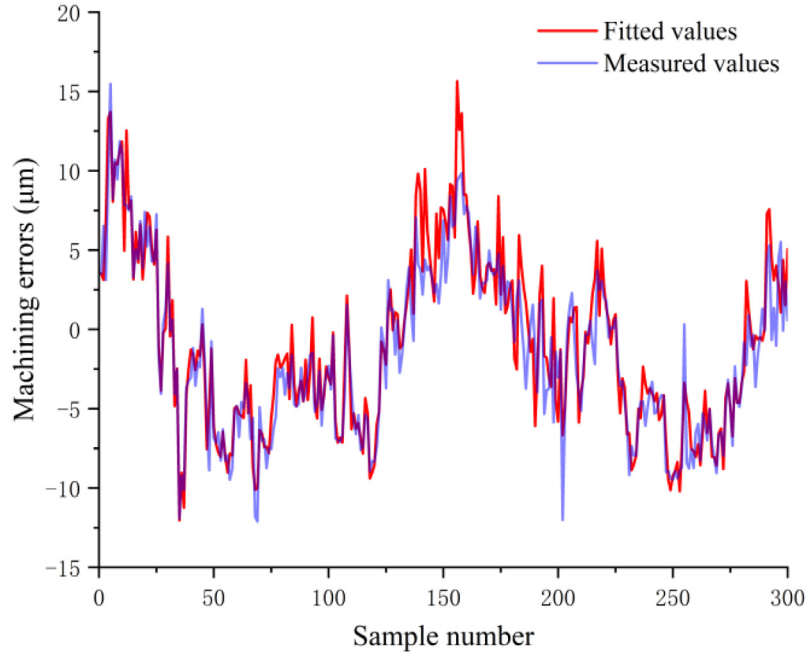


Fig. 9. Fitting result of metabolic GM(1, N) model ($N=9$).

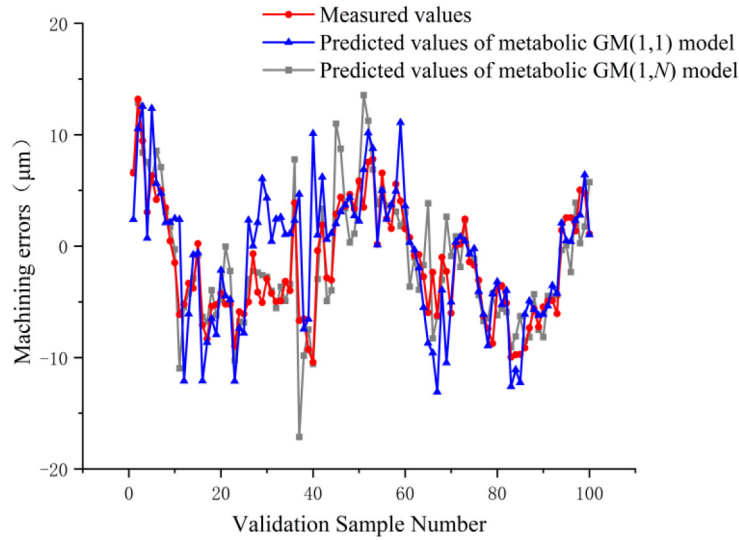


Fig. 10. Prediction results of different models.

4.3 Correction of predicted values by Markov models

The residual values of the prediction results of the metabolic GM(1, N) model were calculated, and then the Markov prediction model was used to correct and optimize the prediction results of the metabolic GM(1, N) model to make up for the shortcomings of the GM(1, N) model and improve the accuracy of the prediction. The absolute values of the residuals before and after the correction are shown in Figure 11.

The prediction results of the metabolic GM(1,1) and the metabolic GM(1, N) prediction models were calculated and analyzed. In this study, the mean absolute error (σ_{MAE}), root mean square error (σ_{RMSE}) and mean absolute percentage error (σ_{MAPE}) were used as evaluation indicators for the analysis. These are defined as follows.

Mean absolute error, denoted by σ_{MAE} ,

$$\sigma_{MAE} = \frac{1}{n} \sum_{i=1}^n |\hat{e}_i - e_i|. \quad (18)$$

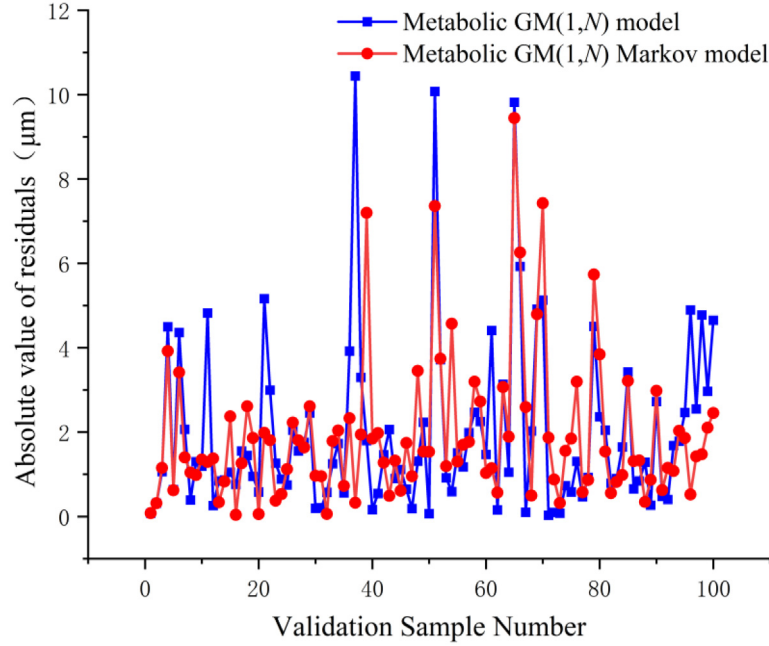


Fig. 11. Absolute values of residuals of predicted results before and after correction.

Table 1. Comparison of prediction accuracies of different models.

Types of models	σ_{MAE} (mm)	σ_{RMSE} (mm)	σ_{MAPE} (%)
Metabolic GM(1,1) model	0.0028	0.0042	92.35
Metabolic GM(1,N) model	0.0020	0.0028	77.49
Metabolic GM(1,N) Markov model	0.0019	0.0026	25.88

Root mean square error, denoted by σ_{RMSE} ,

$$\sigma_{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (\hat{e}_i - e_i)^2}. \quad (19)$$

Mean absolute percentage error, denoted by σ_{MAPE} ,

$$\sigma_{MAPE} = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{\hat{e}_i - e_i}{e_i} \right|. \quad (20)$$

Here n is the validation sample size, $\hat{e}_i$ is the predicted value of the validation sample machining error and e_i is the actual measured value of the validation sample machining error. The more the above three indicators converge to 0, the better the prediction is. The results of the analysis are shown in Table 1.

The metabolic GM(1,N) Markov model proposed in this study was compared with the metabolic GM(1,1) model and the metabolic GM(1,N) model for free-form surface machining error prediction results, and the σ_{MAE} , σ_{RMSE} and σ_{MAPE} values of each model were calculated separately. As shown in Table 1, when the predictions using the metabolic GM(1,N) model were compared with the predictions using the metabolic GM(1,1) model, the

root mean square error of the metabolic GM(1,N) model decreased by 0.0014 mm, and the mean absolute percentage error decreased by 14.86%. When predictions using the metabolic GM(1,N) Markov model were compared with predictions using the metabolic GM(1,N) model, the mean absolute percentage error decreased by 51.61%. The prediction accuracy of the metabolic GM(1,1) model could be further improved by the metabolic GM(1,N) Markov model. For the prediction of the free-form surface machining errors, the prediction of the metabolic GM(1,N) model can be greatly improved by adapting the metabolic GM(1,1) model, and the prediction accuracy of the model can be improved by adding Markov theory to further optimize the model.

5 Conclusion

In this study, the prediction of free-form machining errors was studied. In response to the insufficient prediction accuracy of the GM(1,1) model, a GM(1,N) prediction model was established, incorporating the idea of metabolism, which effectively improved the prediction accuracy. On this basis, the prediction results of the metabolic GM(1,N) model were revised using a Markov prediction model,

and the experimental results showed that the combination of metabolic theory, a grey model, and Markov theory prediction could further improve the prediction accuracy.

Using Markov's principle to correct the fitting results of the metabolic GM(1,N) model could effectively reduce the impact of random fluctuations of the free-form machining errors on the grey model and enhance the adaptability of the grey prediction model in machining error model prediction. This method has practical implications for extending the use of grey models in mechanical engineering. When measuring machining errors on free-form surfaces, the model can be used to predict machining errors at other locations on the workpiece more accurately with only a small amount of measurement point information, saving time and improving efficiency.

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Conflicts of interest

The authors declare no conflict of interest.

Data availability statement

The datasets generated or analyzed during this study are available from the corresponding author on reasonable request.

Author contribution statement

Dayang Ma: Conceptualization, Methodology, Formal Analysis, Writing - Original Draft, Data Curation.

Yueping Chen: Conceptualization, Funding Acquisition, Resources, Project Administration, Supervision, Formal Analysis, Methodology, Writing - Review & Editing.

References

1. M. Marek, Selection of the best model of distribution of measurement points in contact coordinate measurements of free-form surfaces of products, *Sensors*. **19**, 5346 (2019)
2. D.Z. Zhao, W.H. Wang, J.Z. Zhou et al., Measurement point sampling method for inspection of parts with free-form surfaces, *Adv. Mech. Eng.* **10**, 1–12 (2018)
3. W. Wolovich, H. Albakri, H. Yalcin, The precise measurement of free-form surfaces, *J. Manuf. Sci. Eng.* **124**, 326–332 (2002)
4. S.F. Liu, M.L. Zhou, J.P. Wang, Research on variation model for predicting machining error of milling surface, *Manuf. Technol. Machine Tool*. **4**, 80–84 (2018)
5. L.X. Gong, J.H. Jiang, A BP neural networks error predictive model of mechanical processing based on L-M algorithm, *Machine Tool Hydraul.* **11**, 67–71 (2013)
6. Q. Zhang, H.X. Wen, Study on the optimization of machining error gray forecast modeling, *Machine Tool Hydraul.* **36**, 61–63 (2008)
7. S.H. Qi, Y.T. Fei, Prediction of error in manufacturing based on the gray model theory, *Trans. Chin. Soc. Agric. Machinery*. **32**, 89–91+94 (2001)
8. H. Sun, F.Y. Peng, L. Zhou et al., A hybrid driven approach to integrate surrogate model and Bayesian framework for the prediction of machining errors of thin-walled parts, *Int. J. Mech. Sci.* **192**, 106–111 (2021)
9. B. Wang, Predicting machining errors by artificial neural network, *J. Jiangsu Univ. Sci. Technol.* **4**, 44–48 (1997)
10. K.Q. Huang, Y.P. Chen, Y.K. Zhang, Machining errors prediction of free-form surfaces: BP neural network algorithm optimized by simulated annealing algorithm, *J. Guangxi Univ. Sci. Technol.* **33**, 69–73+82 (2022)
11. D.Y. Wu, P. Yan, Y. Guo et al., A gear machining error prediction method based on adaptive Gaussian mixture regression considering stochastic disturbance, *J. Intell. Manuf.* **2021**, 1–19 (2021)
12. H. Manikandan, T.C. Bera, Modelling of dimensional and geometric error prediction in turning of thin-walled components, *Precis. Eng.* **72**, 382–396 (2021)
13. X.L. Yang, M. Zhou, B. Zeng, A new method for constructing background value of grey prediction model, *Statistics & Decision*. **19**, 14–18 (2018)
14. M.X. Shen, T. Tao, Research on thermal error compensation of milling machine based on improved grey network, *Mach. Design Manuf.* **4**, 158–161 (2021)
15. G.D. Li, S. Masuda, D. Yamaguchi et al., A study on the prediction of machining accuracy, *Int. J. Adv. Manuf. Technol.* **43**, 5–6 (2009)
16. W. Zhou, X. Zhu, J. Wang et al., New error prediction method for machining process based on a combined model, *Math. Probl. Eng.* **2018**, 1–8 (2018)
17. Y.Y. Zhao, H.L. Lin, D.H. Zhang et al., Forecasting of grassland carrying capacity of Gannan based on Grey-Markov residual error models, *Trans. Chin. Soc. Agric. Eng.* **28**, 199–204 (2012)
18. L. Yang, H.F. Ning, J. Gong, Research on turning dimension prediction based on grey weighted Markov combined model, *Mach. Design Manuf.* **8**, 39–42+45 (2016)
19. L.L. Zou, M. Cong, Y.R. He, Machining error prediction based on Grey-Markov theory, *Modular Mach. Tool Automatic Manuf. Technique*. **1**, 14–16+19 (2014)
20. C.W. Lu, H. Zhao, S. Jiang et al., A prediction method of rail transit passenger volume based on metabolism – GM(1,N)

- Markov model, J. Highway Transp. Res. Dev. **35**, 141–150 (2018)
21. E.H.K. Fung, Y.K. Wong, H.F. Ho, Modelling and prediction of machining errors using ARMAX and NARMAX structures, Appl. Math. Modell. **27**, 611–627 (2003)
22. J. Teng, Z.J. Xia, M.Y. Lou, Dynamic network public opinion crisis prediction based on metabolic GM(1,N) Markov model, Inf. Sci. **38**, 88–94 (2020)
23. A. Wang, Y. Yang, A combined gray forecast model based on least square method and its application, Math. Practice Theory. **48**, 291–299 (2018)
24. Y.P. Chen, N.Q. Shang, Comparison of GA, ACO algorithm, and PSO algorithm for path optimization on free-form surfaces using coordinate measuring machines, Eng. Res. Express. **3**, 4 (2021)

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